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# To dissent or not to dissent? Informative dissent and parliamentary governance

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**Abstract** Legislative dissent has detrimental effects for both party and legislator, i.e., legislators depend on their party label for re-election, which value in turn depends in part on the party's reputation of cohesiveness. Commonly dissent has been attributed to "extreme" preferences. I provide an informational rationale for dissent. Costly dissent allows the legislator to credibly signal information about his constituency's preferences to the Cabinet. As a result the Cabinet can better calibrate its policies with the electorate's preferences. Dissent is shown to depend on policy preferences as well a the legislators' electoral strength, electoral volatility, and the cost of dissent. Finally, the results suggests that parties may sometimes benefit from tolerating some level of dissent.

**Keywords** Dissent · Parliamentary government · Intra-party politics · Cabinets

**JEL Classification** C72 · D72 · D82

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## 1 Introduction

Parliamentary systems are generally characterized by a high degree of party cohesion in legislative voting. In parliamentary systems the cabinet must retain the confidence of parliament. Losing a vote in parliament is often construed as a sign that the cabinet no longer has the confidence of parliament, which may trigger a vote of no confidence or simply embarrass the cabinet. In some countries, the government steps down after losing a vote on a major bill.[1] Government parties, therefore, have a strong incentive to ensure that their members vote the party line in parliament.

Although party cohesion in parliamentary systems is high it is not perfect. Norton (1975, 1978, 1985, 1987) and Crowe (1980), for example, have demonstrated that cross-voting in the British parliament is fairly frequent, and occasionally, even disruptive of the government's program.[2] Cross-voting is important, not just because it signals that the government lacks support among its own members, but also because it raises questions among voters, first, about whether the party is capable of delivering on its policy promises, and second, about what policies the party actually stands for. Backbenchers, of course, have access to more subtle forms of dissent; the most important one being to voice their policy concerns. Vocally objecting to party policy does not directly interfere with the government's legislative agenda, or threaten its survival in the way that cross-voting does, but if it is done publicly (in the media or the House rather than in caucus, for example), it still places at risk the party's reputation as a coherent and effective agent of government.

The literature has neither satisfactorily answered the question why dissent and, in particular, cross-voting occurs nor considered its implications for policy-making. A simple explanation might posit that parliamentarians dissent if they find the cabinet's policy sufficiently distasteful. This explanation is wanting, however, since MPs in many parliamentary systems are highly dependent on their party labels for re-election, and the party for advancement within its ranks (Carey and Shugart 1996). If dissent risks the party's reputation for unity (Saalfeld 1995), and if that reputation is a valuable electoral asset (Denver 1998; Montgomery 1999; Palmer 1995), then both the party and the MP suffer from dissent (Crowe 1980). Why MPs dissent is thus a bit of a puzzle with few explanations in the literature.

Certainly, the implicit assumption in the literature is that dissent is driven by preference outliers—rather than a desire to broaden the party's coalition or electoral appeal.[3] This is an assumption that I do not argue with. Indeed, in the model presented below I show the existence of an equilibrium that is consistent with this assumption. The

[1] In most instances, Ireland being the exception, this is a convention and not a constitutional rule. See, e.g., De Winter (1995).

[2] Cross-voting involves a member of one party voting with an opposing party in a division (roll-call vote).

[3] In addition to arguing that the desire to influence policy is a motivation for dissent, Crowe (1980) and Mughan (1990) argue that constituency pressure may also be an important factor. Lucy (1993) goes a step further arguing that parties tolerate dissent by MPs in constituencies where a particular policy is unpopular.

model also allows me to examine how the MPs' decisions to dissent is influenced by the electoral costs of dissent, the MPs' electoral strength in their districts, the electoral strength of their party, and the extent to which the voters in the MPs' constituencies hold them accountable for the Cabinet's policy decisions.

I assume that the Cabinet and the MPs are motivated by re-election concerns. While the Cabinet and the MPs' interests tend to go together, i.e., both prefer that the MP retains his seat, they are not perfectly aligned. The MP would like the Cabinet to implement the policy that maximizes his chance of re-election, i.e., the preferred policy of the median voter in the MP's constituency.[4] The Cabinet, on the other hand, would like to retain control of parliament and seeks to maximize its number of MPs. As the Cabinet cannot satisfy the wishes of all its MPs when the constituencies are ideologically heterogenous, it must find the policy compromise that maximizes the likelihood of retaining a majority in parliament. However, this decision is not trivial if the Cabinet has incomplete information about the median voters' preferences in each constituency (i.e., the MP's ideal policy).

The Cabinet must rely on its MPs, who are better attuned to their constituents' preferences, in order to implement the policy that maximizes its probability of retaining a parliamentary majority. However, because the Cabinet and the MPs's preferences over policy diverge, the MPs face an incentive to misrepresent the preferences of their constituents. By claiming that his constituents' preferences are relatively extreme, the MP may hope to induce the Cabinet to find a compromise that gives undue weight to the preferences of his constituents. The problem is further compounded by the fact that all MPs face similar incentives, thus, driving the MPs to signal every more extreme policy preferences. The consequence is that, realizing what incentives the MPs face, the Cabinet will regard the MPs' messages as cheap talk.

Thus, while in principle, MPs can engage in "private" dissent, i.e., behind closed door, such dissent is unlikely to be costly to the MPs and will, therefore, be regarded as cheap talk. However, if MPs dissent publicly, e.g., by voting against or publicly denouncing the Cabinet's proposal, their signals will regain a degree of credibility, allowing them to transmit information about their constituencies to the Cabinet.

Accordingly, dissent is modeled here as a costly action. Facing a Cabinet proposal, each MP decides whether to dissent or to toe the line. Public dissent imposes a cost on all the party's MPs as the party's reputation suffers from dissent. Observing the MPs' decisions, the Cabinet can update its beliefs about the preferences of the MPs' constituents and revise its policy as to maximize its probability of retaining a parliamentary majority.

Recent work on party governance have considered the effects of dissent or intra-party disagreements. Caillaud and Tirole (2002) argue that the possibility of intra-party disagreement enhances parties' electoral prospects but, counter to the findings presented

[4] It is possible that the MP's intended audience is not his constituency but the whole nation, e.g., if the MP is vying for a leadership position within the party or Cabinet. While this scenario is plausible for some established MPs, it is unlikely for the vast majority of MPs as advancement in parliamentary systems tends to regulated by the party organization.

below, that actual expressions of disagreement hurt the party. In an extension of Caillaud and Tirole's model, Castanheira et al. (2005) find that the possibility of disagreement is beneficial when voters are relatively uninformed about the candidates' performance and when the perks of office are low. Beniers (2005) examines a model in which party leaders' ability to fire legislators influences dissent but that such ability leads to worse policies if the party leader is incompetent.

While the above literature argues that dissent often has negative effects, the literature on organizational dissent has shown that dissent may improve organizational performance. In Landier et al. (2006), e.g., (uninformative) dissent in the chain of command can induce decision makers to prioritize information over their own personal preferences and, subsequently, generate more trust among those invested with implementing their decision that the correct (or socially optimal) proposal has been chosen. Argyres and Mui (2005) consider a situation where dissent may be informative, as is the case in this paper, but is subject to the problem of cheap talk because the agent obtains private benefits from dissent, which are analogous to the MPs' re-election motives in the present paper. Argyres and Mui (2005) show that dissent can be informative under the appropriate rules of engagement, which essentially make dissent (more) costly.

The model below builds on the insights offered in the literature on parliamentary dissent and shows that costly dissent aids information transmission. The model, thus, carries the analysis a step further to enrich our understanding of, and to generate additional hypotheses about, the Cabinet and its MPs' behavior. In particular, while the literature on parliamentary dissent simply asserts that MPs representing constituencies with relatively extreme policy preference will dissent, the decision to dissent is endogenously determined in the model considered here and the MPs and the Cabinet's strategies are shown to depend on the MPs' electoral strength and the cost of dissent.

Section 2 presents the model and shows the existence of an equilibrium in which extremism is associated with dissent when the MPs' decision rule is endogenously determined. Section 3 offers some insights into the characteristics of the equilibrium when more specific assumptions are made about the players' utility functions. The model does not easily allow characterization of the equilibrium so I look for equilibria computationally. By looking at a number of different combinations of parameter values, I show how dissent is influenced by the cost of dissent, the MPs' electoral strength, the parties' electoral strength, and the extent to which the voters hold their MP accountable for their Cabinet's policy choices. I also consider the effect of costly dissent on policy and the Cabinet's probability of winning re-election. The final section summarizes the results and suggests avenues for further research.

## 2 A model of parliamentary dissent

My model of parliamentary dissent focuses on the interaction between three players; the Cabinet, $C$, and two MPs, labeled $i$ and $j$. Each player has a preference over policy outcomes in an uni-dimensional policy space.

The game has three stages. The sequence of play is depicted in Fig. 1. At the first stage nature draws the MPs' types independently from an uniform distribution,

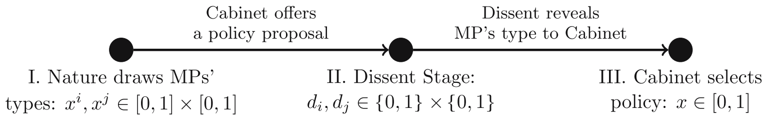


**Fig. 1** The sequence of play

$x^k \sim U[0, 1]$, $k \in \{i, j\}$.[5] For convenience I will occasionally use the terms "left" (or "liberal") and "right" (or "conservative") to refer to the candidates' types (with "left" and "liberal" corresponding to a lower $x^k$). Let $F_x$ be the cumulative density function and $f_x$ be the associated density function. The MPs', or their constituencies', ideal points are private information, i.e., only the MP knows his ideal policy with certainty. At the second stage the MPs decide whether to dissent or not, $d_k \in \{0, 1\}$, where $d_k = 1$ indicates dissent. Dissent is assumed to perfectly reveal the MP's type to the Cabinet.[6] As dissent is costly for the cabinet party's reputation, the re-election probabilities of *both* MPs are reduced by $\alpha$. The MPs' beliefs, as well as the Cabinet's, are described below. At the final stage, upon observing the MPs' actions, the Cabinet selects a policy, $x \in [0, 1]$, as to maximize the probability of having both MPs reelected.[7]

Note that dissent occurs before the Cabinet takes any action in the model. Explicitly modeling the Cabinet's initial policy proposal has no consequence for the results obtained below because only the policy that is implemented at the last stage of the game influences the MPs' re-election probabilities. That is, the decision to dissent is anticipates the Cabinet's policy choice. The fact that a decision looms on the horizon is equivalent to having a Cabinet proposal on the table.[8]

The MPs want to maximize their probability of winning re-election. The closer the Cabinet's policy is to the median voter's ideal point in the MP's constituency, the more likely the MP is to be re-elected. The MP's probability of re-election equals:

$$r_k(x) = \max \left\{ 0,\ p_k + u(x, x^k) - \alpha \sum_{l \in \{k, \sim k\}} d_l \right\} \tag{1}$$

[5] The uniform function is chosen for its simplicity. Other functional forms might provide a better fit for the distribution of MPs' types but the results would remain qualitatively the same although the quantitative predictions of the model might change.

[6] This assumption is at odds with the discussion in the introduction since the MP still has an incentive to misrepresent his preferences. Substantively the results remain the same if the more reasonable assumption that the Cabinet's uncertainty is merely reduced following the MP's decision to dissent or not. The derivation of the results is considerably less cumbersome algebraically when the simplifying assumption is adopted.

[7] Cowley (2002), Crowe (1980), and Mitchell (1999), for example, cite instances where dissent has resulted in modification of government policies.

[8] Although allowing the Cabinet to propose a bill before the MPs dissent might appear to give the Cabinet an opportunity to behave strategically by offering a proposal that induces dissent from the MPs—and thereby learn something about there preference—this possibility is ruled out by the fact that the Cabinet cannot commit to a proposal different from 0.5 in the event no dissent occurs. Since the optimal decision in the event of no dissent forms the only credible proposal the proposal itself will have no effect on the MPs decision calculus.

where $p_k \in [0, 1]$ denotes the MP's electoral strength defined as his probability of re-election if no dissent occurs and the Cabinet implements his constituency's ideal policy. The function $u(x; x^k) = u_k(x)$ is strictly concave, twice differentiable, and symmetric about its maximum, $x^k$, the MP's ideal point. The function $u(x; x^k)$ can equivalently be represented as $w(x^k - x)$, i.e., the MPs' functions only differ in the location of their maxima. It is assumed that the function takes the value zero at its maximum, i.e., $u(x^k; x^k) = 0$. The final term in the re-election probability function represents the cost of dissent, $\alpha \in \mathbb{R}^+$, and the MPs' decision to dissent, $d_k \in \{0, 1\}$. As discussed above, if $\text{MP}_i$ dissents, the reputation of the party is damaged, and both $\text{MP}_i$ and $\text{MP}_j$'s re-election probabilities are reduced by $\alpha$. It is also assumed that $p_k$ is relatively large, so that $r_i(x) > 0$ and $r_j(x) > 0$, $\forall x \in [0, 1]$. This avoids the complexity that stem from the non-concavity of $r_k$, caused by the requirement that the re-election probabilities lie in the interval [0, 1].[9]

The Cabinet is concerned with maintaining its majority in parliament, i.e., the electoral fortune of the party as a whole. The Cabinet seeks to maximize the probability of having both MPs reelected: $r_C(x) = r_i(x) * r_j(x)$. This is the exact probability of having both MPs re-elected.[10] Other things equal the party frontbench would prefer a larger majority (minority) to a smaller one but the all-important question is whether the party can form a government.[11] Since it is assumed that the Cabinet has incomplete information about the MPs' preferences, before any dissent occurs, the Cabinet seeks to maximize:

$$r_C(x) = E\left[\max\left\{0, p_i + u_i(x) - \alpha \sum_{l \in \{i,j\}} d_l\right\}\right] * E\left[\max\left\{0, p_j + u_j(x) - \alpha \sum_{l \in \{i,j\}} d_l\right\}\right]$$

[9] The assumption is warranted by the fact that the MPs, by definition, won their last election, i.e., their probability of re-election is likely to be fairly large. In addition, it is reasonable to assume that the Cabinet will primarily be concerned with MPs that have a chance of re-election.

[10] One may wonder whether this probability cannot be approximated by a simpler function, i.e., $r_C(x) = r_i(x) + r_j(x)$. In short, such simplification comes at a substantial cost as it implies that the MPs' prior re-election probabilities have *no* influence on the Cabinet's policy choice and, subsequently, the likelihood of dissent.

[11] Three caveats are in order. First, more generally, the Cabinet's problem is that of maximizing the probability of re-electing at least $x$ of $n$ MPs, where $x \leq n$. In some circumstances it is rational for the Cabinet to focus its attention on a subset of its MPs. Second, there are constituencies represented by the opposition. For the purpose of studying dissent the importance of these potential candidates is minimal because their scope for dissent is limited, and presumably, on average their chance of winning is lower than that of elected MPs. Third, where coalition governments are the norm it is reasonable to assume plurality maximization. Each of these factors would add a layer of complexity to the model but they do not alter the basic characteristics of the Cabinet's problem—in each of these scenarios the Cabinet must balance the electoral fortunes of some MPs against others, which in turn provides the MPs with differing incentives to dissent as shown below.

The expectations reflect the fact the Cabinet does not know the location of the MPs' ideal points with certainty.

## 2.1 The cabinet's strategy

Proceeding by backwards induction, consider the Cabinet's strategy. The MPs are assumed to adhere to a *symmetric cut-off* strategy—they dissent if they find themselves "sufficiently" far away from the center of the policy space. A *symmetric cut-off* strategy refers to the two cut-off points, on the left and the right, being equidistant from the center of the policy space. Formally, $\text{MP}_k$ dissents if $x^k \in [0, \underline{x}_k] \cup [\overline{x}_k, 1]$, and does not dissent if $x^k \in (\underline{x}_k, \overline{x}_k)$.

Given the MPs' cut-off strategies, the Cabinet can update its beliefs about the MPs' types. The Cabinet's prior belief about $\text{MP}_k$ being of type $x^k$ is $b_C(x^k) = f_x$. As the Cabinet knows the distribution from which the MPs' types are drawn, the Cabinet updates its beliefs in a simple Bayesian fashion after observing whether the MPs dissent. If $\text{MP}_k$ dissents, $b_C(x^k) = 1$. If $\text{MP}_k$ does not dissent, the Cabinet learns that the MP's ideal point must lie between the two cut-off points. By Bayes' rule the Cabinet's beliefs are:

$$b_C(x^k) = \begin{cases} \frac{f_x}{\overline{x}_k - \underline{x}_k} & \text{if} \quad x^k \in [\underline{x}_k, \overline{x}_k] \\ 0 & \text{else} \end{cases}$$

Solving for the Cabinet's strategy involves considering four situations that the Cabinet may face. The subgames differ in terms of the uncertainty facing the Cabinet and this in turns influences his optimal choice of action. The four scenarios that the Cabinet may face are: (i) Neither MP dissents, (ii) only $\text{MP}_i$ dissents, (iii) only $\text{MP}_j$ dissents, and (iv) both MPs dissent.

Consider the case when neither MP has dissented. The Cabinet's expected utility is:

$$r_C(x) = \left[ p_i + \int_{\underline{x}_i}^{\overline{x}_i} u(x; x^i) \frac{1}{\overline{x}_i - \underline{x}_i} dx^i \right] * \left[ p_j + \int_{\underline{x}_j}^{\overline{x}_j} u(x; x^j) \frac{1}{\overline{x}_j - \underline{x}_j} dx^j \right] \quad (2)$$

The integration over the intervals $[\underline{x}_i, \overline{x}_i]$ and $[\underline{x}_j, \overline{x}_j]$ reflects that neither MP dissented, i.e., given the MPs' cut-off rule, the Cabinet can update its beliefs about the location of the MPs' ideal points. The Cabinet does best choosing $x = 0.5$. To see this it is sufficient to show that the expected re-election probability of each MP is maximized at 0.5, i.e., that $E[u(0.5; x^k)|d_i = d_j = 0] \geq E[u(x; x^k)|d_i = d_j = 0], \forall x \in [0, 1]$. By strict concavity of $u(x; x_k)$, $E[u(x; x_k)]$ has an unique maximum, $x^*$, characterized by the first order condition:

$$\left. \frac{dE\left[u(x; x^k)|d_k = 0\right]}{dx} \right|_{x=x^*} = \int_{\underline{x}_k}^{\overline{x}_k} \frac{dw(x^k - x)}{dx} f(x^k) dx^k \bigg|_{x=x^*} = 0 \quad (3)$$

where $f(x^k)$ is a symmetric density function about 0.5. Lemma 1, proven in the appendix, shows that this condition is satisfied at $x = 0.5$ and one can then conclude, by strict concavity, that $x^* = 0.5$.

**Lemma 1** *If $f(x)$ is a symmetric density function about 0.5 then $r_C(x) = \left[p_i + \int_{\underline{x}_i}^{\overline{x}_i} u(x; x^i) f(x^i) dx^i\right] * \left[p_j + \int_{\underline{x}_j}^{\overline{x}_j} u(x; x^j) f(x^j) dx^j\right]$ is maximized at $x = 0.5$.*

Lemma 1 states that when neither MP dissents, the Cabinet can do no better than choose the midpoint of the distribution of the MPs' types. Intuitively, the midpoint minimized the expected distance between the Cabinet's choice and the MP's ideal policy.

In the second and third scenarios, when only one MP dissents, the Cabinet learns his policy preferences but remains uncertain about the other MP's preferences. Let $\text{MP}_k, k \in \{i, j\}$, denote the dissenting MP and $\text{MP}_{\sim k}$ the non-dissenter. The Cabinet's expected utility can then be written as:

$$r_C(x) = \left[p_{\sim k} + \int_{\underline{x}_{\sim k}}^{\overline{x}_{\sim k}} u(x; x^{\sim k}) \frac{1}{\overline{x}_{\sim k} - \underline{x}_{\sim k}} dx^{\sim k} - \alpha\right] * \left[p_k + u(x; x^k) - \alpha\right] \quad (4)$$

Taking the first derivative yields:

$$\frac{\partial r_C}{\partial x} = \int_{\underline{x}_{\sim k}}^{\overline{x}_{\sim k}} \frac{\partial u(x; x^{\sim k})}{\partial x} * \frac{1}{\overline{x}_{\sim k} - \underline{x}_{\sim k}} dx^{\sim k} * \left[p_k + u(x; x^k) - \alpha\right]$$
$$+ \frac{\partial u(x; x^k)}{\partial x} * \left[p_{\sim k} + \int_{\underline{x}_{\sim k}}^{\overline{x}_{\sim k}} u(x; x^{\sim k}) \frac{1}{\overline{x}_{\sim k} - \underline{x}_{\sim k}} dx^{\sim k} - \alpha\right] \quad (5)$$

By the concavity of $r_k$ and $r_{\sim k}$ in $x$, the Cabinet has an unique best response denoted $x^{*k}$ where the superscripted $k$ denotes dissent by $\text{MP}_k$). A sketch of the proof of uniqueness follows the discussion of the last scenario. The terms outside the brackets in Eq. (5) represent the (expected) marginal change in each of the MPs' re-election probabilities whereas the terms inside the brackets are the MPs' re-election probabilities. This reflects the fact that the Cabinet faces a trade-off in improving its chances of retaining parliamentary majority. The value of a marginal increase in $\text{MP}_{\sim k}$'s re-election probability depends on $\text{MP}_k$'s re-election probability.

In the last scenario both MPs dissent. Label the Cabinet's best response in this scenario as $x^{*ij}$. I show that the Cabinet's best response is unique in this scenario. The Cabinet's best response in the other scenarios can be shown to be unique in an analogous manner by substituting the expected utilities into the equations below. First, note that the Cabinet's best response must lie between the MPs' ideal policies. Then, assuming $x^i < x < x^j$, the marginal change in the re-election probability from an increase in $x$ is negative for $\text{MP}_i$, $\frac{\partial u_i(x)}{\partial x} < 0$, and positive for $\text{MP}_j$, $\frac{\partial u_j(x)}{\partial x} > 0$. The Cabinet's best response is found by setting the f.o.c. equal to zero:

$$\frac{\partial r_C}{\partial x} = \frac{\partial u_i(x)}{\partial x} * \left[p_j + u_j(x) - 2\alpha\right] + \frac{\partial u_j(x)}{\partial x} * \left[p_i + u_i(x) - 2\alpha\right] = 0 \quad (6)$$

Rearranging the terms:

$$\frac{p_i + u_i(x) - 2\alpha}{p_j + u_j(x) - 2\alpha} = -\frac{\frac{\partial u_i(x)}{\partial x}}{\frac{\partial u_j(x)}{\partial x}} \quad (7)$$

The term on the LHS of (7) is clearly decreasing in $x$ in the relevant region (i.e. between the MPs' ideal points), whereas the term on the RHS is increasing in $x$. Hence, (7) has an unique solution if one exists. If the solution to (7) is not in the interval $[x^i, x^j]$, the Cabinet's best response corresponds to whichever of the MPs' ideal points that is closer to the solution. It also follows from (7) that if the MPs' prior re-election probabilities are equal and both MPs dissent, the Cabinet will set its policy at the midpoint between the MPs' ideal points, $\frac{x^i + x^j}{2}$.

It is worth noting that dissent has the intended effect from the MP's point of view (when the other MP does not dissent), i.e., by dissenting the MP obtains a more favorable policy outcome than if he had toed the party line. That is, if $\text{MP}_i$ dissents and $x^i > \bar{x}_i$ then the Cabinet will implement a policy further to the right than if no dissent had occurred, $x^* > \frac{1}{2}$. To see why this is the case, note that when a MP does not dissent, the Cabinet's maximized the expected utility of the MP by choosing $x = 0.5$ (see Lemma 1). The marginal effect of the Cabinet's policy on the non-dissenting MP's utility evaluated at $x = 0.5$ equals zero at this point, i.e., $\left(\frac{\partial E[U_j]}{\partial x}\Big|_{x=0.5} = 0\right)$. However, since $\text{MP}_i$ dissents and $x^i > \bar{x}_i$, $\text{MP}_i$ strictly benefits from a move in the policy rightwards, i.e., $\left(\frac{\partial U_i}{\partial x}\Big|_{x=0.5} > 0\right)$. Thus, choosing a policy at, or to the left of, 0.5 cannot be optimal for the Cabinet.

It is also worth considering how the MPs' prior probabilities of re-election and the cost of punishment influence the Cabinet's best response. W.l.o.g., consider the case when both MPs dissent. Implicitly differentiating (6) with respect to $p_i$ yields the marginal change in the Cabinet's policy:

$$\frac{\partial x^{*ij}}{\partial p_i} = -\frac{\frac{\partial u_j(x^{*ij})}{\partial x}}{\frac{\partial^2 u_i(x^{*ij})}{\partial x^2}\left[p_j + u_j(x^{*ij}) - 2\alpha\right] + 2\frac{\partial u_i(x^{*ij})}{\partial x}\frac{\partial u_j(x^{*ij})}{\partial x} + \frac{\partial^2 u_j(x^{*ij})}{\partial x^2}\left[p_i + u_i(x^{*ij}) - 2\alpha\right]} \quad (8)$$

The marginal change in the Cabinet's policy, $\frac{\partial x^{*ij}}{\partial p_i}$, is positive in the interval $[x^i, x^j]$ by virtue of the concavity of $u_k(x)$ and by the sign of the first derivatives of $u_k(x)$ in the interval. The marginal effect of a change in $p_j$ is negative—one need only replace the numerator in (8) by $\frac{\partial u_i(x)}{\partial x}$. Hence, an increase in a MP's prior re-election probability reduces the extent to which the Cabinet is willing to accommodate his policy preferences. That is, marginal MPs are better able to extract policy concessions from the Cabinet than MPs in relatively safe seats.

To consider the effect of a change in the cost of dissent on the Cabinet's policy choice, implicitly differentiate (6) with respect to $\alpha$:

$$\frac{\partial x^{*ij}}{\partial \alpha} = \frac{\frac{\partial u_j(x^{*ij})}{\partial x} + \frac{\partial u_i(x^{*ij})}{\partial x}}{\frac{\partial^2 u_i(x^{*ij})}{\partial x^2}\left[p_j + u_j(x^{*ij}) - 2\alpha\right] + 2\frac{\partial u_i(x^{*ij})}{\partial x}\frac{\partial u_j(x^{*ij})}{\partial x} + \frac{\partial^2 u_j(x^{*ij})}{\partial x^2}\left[p_i + u_i(x^{*ij}) - 2\alpha\right]} \tag{9}$$

The sign of (9) depends on the relative magnitudes of $\frac{\partial u_j(x^{*ij})}{\partial x}$ and $\frac{\partial u_i(x^{*ij})}{\partial x}$, which are equal in equilibrium unless the MPs' prior probabilities are unequal. If the prior probabilities are equal, the Cabinet chooses the midpoint between the MPs' ideal points and the cost of dissent doesn't influence the Cabinet's decision. If $\text{MP}_i$ has a higher prior probability of re-election then $\frac{\partial u_j(x^{*ij})}{\partial x} < -\frac{\partial u_i(x^{*ij})}{\partial x}$, in which case the numerator of (9) is negative and $\frac{\partial x^{*ij}}{\partial \alpha} > 0$.[12] Hence, safer MPs become increasingly disadvantaged as the cost of dissent increases.

As the Cabinet seeks to maximize the probability of having both MPs re-elected, the marginal value of a policy concession to a secure MP is diminished by the fact it is effectively discounted by the low re-election probability of the marginal MP as the likelihood of retaining a majority equals $r_C = r_i r_j$. The Cabinet must balance off the re-election probabilities of the MPs and this favors marginal MPs. The extent to which the more marginal candidate is favored increases as the cost of dissent increases.

These findings runs counter to the view that electorally strong MPs wield more policy influence. That view may confuse correlation with causation, however. Party leaders, who wield greater policy influence, are more likely to be strong electorally. It is, however, their rank within their party, not their electoral strength, that makes them influential. The electoral strength of the party elite may derive from quite different sources such as name recognition and, perhaps, the ability to provide particularistic goods.

It bears noting that the analysis of the Cabinet's best response, i.e., treating the MPs' cut-off strategies as exogenously given, corresponds roughly with the prevalent view of dissent in the empirical literature, i.e., that MPs simply dissent when their constituents are sufficiently unhappy about the Cabinet's policy. The discussion above suggests that this view is wanting. The fact that the Cabinet takes the MPs' electoral strength and the cost of dissent into account obviously influences the benefits that the MP reaps from dissenting. A rational MP's willingness to dissent depends, therefore, on these same factors.

[12] This applies only to the history where both MPs dissent. If only one MP dissents, both differences in the prior re-election probabilities and the uncertainty about the preferences of the non-dissenting MP influence the marginal effect of $\alpha$ on the Cabinet's policy.

## 2.2 The MPs' strategies

The MPs' decision whether to dissent or acquiesce takes account of both the cost of dissenting and the consequences of revealing their policy preferences to the Cabinet. Each MP forms expectations about whether the other MP will dissent. As both the MPs act simultaneously, $MP_k$'s beliefs about $MP_{\sim k}$'s preferences can simply be described as $b_k(x^{\sim k}, \underline{x}_{\sim k}) = f_x$, where $b_k(x^{\sim k}, \underline{x}_{\sim k})$ is the probability that $MP_{\sim k}$'s type is $x^{\sim k}$ given $MP_{\sim k}$'s strategy. This reflects the fact that no information has been revealed and the MPs' knowledge is restricted to the distribution of MPs' types. Given these beliefs, and the MPs' strategies, $MP_k$ can infer that $MP_{\sim k}$ will be a dissenting liberal with probability $\int_0^{\underline{x}_{\sim k}} f_x = \underline{x}_{\sim k}$. Similarly, $MP_{\sim k}$ will be a dissenting conservative with probability $1 - \overline{x}_{\sim k} = \underline{x}_{\sim k}$, and $MP_{\sim k}$ will not dissent with probability $1 - 2\underline{x}_{\sim k}$.

Now consider the MPs' strategies. A cut-off strategy is a pair $(\underline{x}_k, \overline{x}_k)$ such that $MP_k$ dissents if and only if $x^k < \underline{x}_k$ or $x^k > \overline{x}_k$. By continuity of the MPs' utility functions in $x$, the existence of a cut-off strategy implies that there exists—provided that the cost of dissent is not prohibitively high (in which case existence of an equilibrium is trivial)—a policy such that $MP_k$ would be indifferent between dissenting or supporting the Cabinet if $x^k = \underline{x}_k$ or $x^k = \overline{x}_k$. Since cut-off policies are equidistant from the center of the policy space it is sufficient to concentrate on the lower cut-off point, $\underline{x}_k$.

The decisions facing the MPs are identical up to the parametrization of their utility functions. Consider $MP_i$'s decision. First, note that the expected utility of $MP_i$ when $MP_j$ dissents will depend on $MP_j$'s location, i.e., $MP_j$ may either be a liberal or a conservative. Let $x^{*\underline{j}}$ and $x^{*\overline{j}}$ denote the Cabinet's optimal policy when $MP_j$'s ideal policy lies to the left or right, respectively, of the center of the policy space. Solving for the lower cut-off policy for $MP_i$ requires that the following equality be satisfied:

$$\begin{aligned}
&\underline{x}_j \left[ p_i + \int_0^{\underline{x}_j} u_i(x^{*\underline{j}}) \frac{1}{\underline{x}_j} dx^j - \alpha \right] + (1 - \overline{x}_j) \left[ p_i + \int_{\overline{x}_j}^{1} u_i(x^{*\overline{j}}) \frac{1}{1 - \overline{x}_j} dx^j - \alpha \right] \\
&+ (\overline{x}_j - \underline{x}_j) \left[ p_i + u_i(0.5) \right] = \underline{x}_j \left[ p_i + \int_0^{\underline{x}_j} u_i(x^{*i\underline{j}}) \frac{1}{\underline{x}_j} dx^j - 2\alpha \right] \\
&+ (1 - \overline{x}_j) \left[ p_i + \int_{\overline{x}_j}^{1} u_i(x^{*i\overline{j}}) \frac{1}{1 - \overline{x}_j} dx^j - 2\alpha \right] + (\overline{x}_j - \underline{x}_j) \left[ p_i + u_i(x^{*i}) - \alpha \right]
\end{aligned} \tag{10}$$

The dependence of the $x^*$'s, the Cabinet's policy choices, on $x^i$ and $x^j$ is suppressed to save on notation. The LHS of (10) is $MP_i$'s expected utility if the MP supports the Cabinet and the RHS is $MP_i$'s expected utility if he dissents. The first term on the LHS represents $MP_i$'s expected utility if $MP_j$'s ideal point is in the interval $[0, \underline{x}_j]$, i.e., $MP_j$ dissents. The probability that $MP_j$ dissents on the left is $\underline{x}_j$ as the distribution of types is uniform. The second term represents in a similar way the probability that

$\mathrm{MP}_j$ dissents on the right end of the policy space times the expected utility of such dissent. The third term is the probability that $\mathrm{MP}_j$ does not dissent multiplied by the utility of no dissent by either MP. Analogous terms appear on the RHS, only now the Cabinet's choices reflect $\mathrm{MP}_i$'s dissent. Simplifying (10) yields:

$$\underline{x}_j\left[\int_0^{\underline{x}_j}\left[u_i(x^{*i\underline{j}})-u_i(x^{*\underline{j}})\right]\frac{1}{\underline{x}_j}dx^j+\int_{\overline{x}_j}^{1}\left[u_i(x^{*i\overline{j}})-u_i(x^{*\overline{j}})\right]\frac{1}{1-\overline{x}_j}dx^j\right]$$
$$+(\overline{x}_j-\underline{x}_j)\left[u_i(x_i^{*i})-u_i(0.5)\right]=\alpha \tag{11}$$

Equation (11) has a straightforward interpretation. If the expected utility of dissent exceeds the expected utility of acquiescence by more than $\alpha$ the MP is better off dissenting. Each of the terms $u_i(x^*)$ is concave in the MP's ideal point, $x^i$. This may not be immediately obvious since the marginal effect of change in $x^i$ can be disaggregated into direct and indirect effects. The direct effect represents the change in utility that results from simply having a different ideal point while the Cabinet's policy is held constant. The indirect effect is the change in the MP's utility that comes about from the change in the Cabinet's policy in response to the change in $x^i$. When evaluated in the interval $[\underline{x}_i, \overline{x}_i]$ the indirect effect equals zero, $\frac{\partial x^*}{\partial x^i} = 0$, as what the Cabinet does not observe does not influence its strategy. In general, however, the two effects work in the opposite direction as the following lemma shows.

**Lemma 2** *Without loss of generality assume $x^i < x^j$. The Cabinet's response to a change in $x^i$, when $MP_i$ dissents, is positive but less than unity, i.e., $0 \leq \frac{\partial x^*}{\partial x^i} \leq 1$.*

The proof of the lemma is in the appendix.

**Corollary 1** *Without loss of generality assume $x^i < x^j$. The Cabinet's response to a change in $x^j$, when $MP_j$ dissents, is negative but less than unity, i.e., $-1 \leq \frac{\partial x^*}{\partial x^j} \leq 0$.*

Lemma 2 and Corollary 1 show that the Cabinet's policy is weakly monotonic in the MPs' ideal policies but also that the Cabinet's response to more extreme ideal policies are moderate.

Equation (10) must be satisfied for both MPs in equilibrium. In order for a symmetric cut-off equilibrium to exist, it has to be the case that the net benefit of dissent, i.e., the expected utility of dissent minus the expected utility of acquiescence, is greater for MPs whose ideal points are located far away from the center of the policy space. I show that the LHS of (11) is strictly decreasing in $x^k, \forall k = \{i, j\}$, in the interval $[0, 0.5]$ in two steps. Lemma 3 shows that each of the $E[u_i(x^*)]$ functions in (11) is concave, which Lemma 4 consequently uses to drive the result home. The proofs are in the appendix.

**Lemma 3** *The functions $u_k(x^{*ij})$, $u_k(x^{*k})$, and $u_k(x^{*\sim k})$, $k \in \{i, j\}$, are concave in $x^k$.*

**Lemma 4** *The terms* $u_i(x^{*i}) - u_i(0.5)$ *and* $\underline{x}_j \left[\int_0^{\underline{x}_j} \left[u_i(x^{*i\underline{j}}) - u_i(x^{*\underline{j}})\right] \frac{1}{\underline{x}_j} dx^j\right] + (1-\underline{x}_j)\left[\int_{\overline{x}_j}^1 \left[u_i(x^{*i\overline{j}}) - u_i(x^{*\overline{j}})\right] \frac{1}{1-\overline{x}_j} dx^j\right]$ *are strictly decreasing in* $x^i$ *in the interval* $[0, 0.5)$*, and strictly increasing in* $x^i$ *in the interval* $(0.5, 1]$.

As the LHS of (11) is strictly decreasing in $x^i$ in the interval $[0, 0.5)$, and strictly increasing in the interval $(0.5, 1]$, an unique best response is guaranteed for all types, $x^k \in [0, 1]$. Furthermore, by Lemma 4, the MPs' best response can be characterized by a cut-off point rule, i.e., $\text{MP}_k$ will dissent if and only if $x^k \notin [\underline{x}_k, \overline{x}_k]$. That is, the MP's utility of dissent increases the further away from the center of the policy space his ideal policy is. Consequently, his expected utility equals the cost of dissent at a single point each side of the center of the policy space. Each MP's best response function depends on the other MP's action. A change in $\text{MP}_k$'s strategy will influence $\text{MP}_{\sim k}$'s expected payoff, and consequently, his optimal strategy. In equilibrium neither MP wants to alter his strategy, i.e., $\text{MP}_k$'s strategy is the best response to $\text{MP}_{\sim k}$'s best response. Proposition 1 shows that such an equilibrium exists.

**Proposition 1** *There exists a symmetric cut-off point strategy equilibrium.*

*Proof* Since I am only concerned with symmetric strategies, i.e., where $\underline{x}_i = 1 - \overline{x}_i$, it is only necessary to show that an equilibrium cut-off exists in the interval $[0, 0.5]$ by the symmetry of the LHS of (11). Then, a pair of cutoff strategies can be described as $(\underline{x}_i, \underline{x}_j) \in [0, 0.5] \times [0, 0.5]$. Then there exists a best response function, $f : [0, 0.5] \times [0, 0.5] \rightarrow [0, 0.5] \times [0, 0.5]$, that maps every pair of cutoff points into best responses to those cutoff points. In equilibrium the cutoff points map onto themselves, i.e., the function has a fixed point. By Brouwer's theorem such a fixed point exists if $[0, 0.5] \times [0, 0.5]$ is compact and convex, which is clearly the case, and if $f$ is continuous, which is guaranteed by the maximum theorem. Hence, a cutoff point strategy equilibrium exists. □

Proposition 1 alongside with the Lemmas 2–4 offer a basic characterization of the game's equilibrium. MPs with extreme policy preferences dissent but how extreme the MP must be is determined endogenously in the model and depends on the model's parameters. This implies that empirical tests of the effect of extremism on the likelihood of dissent must control for factors such as electoral strength and the cost of dissent. Obtaining more specific predictions about the effects of these variables, and how they interact with the MPs' policy preferences, is desirable.

Proposition 1 shows that the game has a symmetric cut-off equilibrium. That does not, however, imply that the equilibrium is unique. Whether it is unique may depend on the shape of the MPs' utility functions as the Cabinet's policy choice depends on the strategies that the MPs employ and the extent, and direction, of the Cabinet's policy changes as a response to the MP's strategy depends on the functional form. It is, therefore, not possible to obtain general results regarding uniqueness of the equilibrium or the effects of changes in the prior re-election probabilities and the cost of dissent. In the next section I solve computationally for the equilibria of the game assuming a particular functional form for the policy component of the MP's utility function.

## 3 Computational results

Assuming that the actor's utility functions have a particular functional form permits computation of the games equilibria for given values of the game's parameters. Computational methods, thus, allow me to establish some of the equilibrium's properties. The process of looking for the equilibria of the game computationally may fail to identify unstable equilibria. It will, however, identify a stable sample equilibrium if one exists.[13] More than one stable equilibrium may exist. To check whether that is the case one can alter the initial values, or actions, as one searches for an equilibrium.[14]

It can be argued that the stable equilibria of the game are the most relevant. Unstable equilibria, as the name suggest, are vulnerable to small perturbations in the players' strategies, whereas stable equilibria are resistant to such deviations. Thus, empirically one would expect to encounter the stable equilibria rather than the unstable ones.

Looking for the equilibria computationally is clearly not the ideal way to characterize the equilibria of the game. But as the game's equilibrium cannot be characterized analytically it remains the best, if not the only, option. Computational methods allow predictions about how the MPs' equilibrium behavior is affected by their prior re-election probabilities and the cost of dissent.

### 3.1 Quadratic policy preferences

The MP's reelection probabilities are assumed to be quadratic in policy preferences:

$$r_k(x) = p_k - \beta \left(x - x^k\right)^2 - \alpha \sum_{l \in \{k, \sim k\}} d_l \tag{12}$$

This formulation of the reelection probability introduces an additional parameter, $\beta$. The parameter $\beta$ reflects the extent to which the voters hold their MP accountable for the Cabinet's policy decisions. I will refer to $\beta$ as electoral volatility.[15] As before I assume that $r_k(x) > 0, \forall x \in [0, 1], k \in \{i, j\}$.

Finding the equilibrium of the game involves identifying a cut-off for each of the MPs such that he is indifferent, given the other MP's strategy, between dissent and acquiescence. That is, in equilibrium each MP's expectation about the other MP's location and probability of dissent are taken into account in forming a best response. When the available strategies are restricted to cut-off point strategies these expectation take a relatively simple form and $\text{MP}_i$'s cutoff strategy, $\underline{x}_i$, solves Eq. (11).

[13] A sample equilibrium is a equilibrium of the game—"sample" refers to the fact that other equilibria may exist.

[14] See Judd (1998) for the use of computational methods to find Nash equilibria in games with continuous strategy spaces. McKelvey and McLennan (1996) discuss finite games.

[15] The type of electoral volatility discussed here should not be confused with the more common use of the term, i.e., fluctuation in electoral outcomes.

Computing the equilibrium of the game involves finding a pair of cut-off strategies, $(\underline{x}_i, \underline{x}_j)$, such that:

$$\underline{x}_{\sim k}\left[\int_0^{\underline{x}_{\sim k}}\left[u_k(x^{*k\sim k}) - u_k(x^{*\sim k})\right]\frac{1}{\underline{x}_{\sim k}}dx^{\sim k}\right] + \left(1-\underline{x}_{\sim k}\right)\left[\int_{\overline{x}_{\sim k}}^{1}\left[u_k(x^{*k\sim k}) - u_k(x^{*\sim k})\right]\frac{1}{1-\overline{x}_{\sim k}}dx^{\sim k}\right] + (\overline{x}_{\sim k} - \underline{x}_{\sim k})\left[u_k(x^{*k}) - u_k(0.5)\right] = \alpha, \forall k \in \{i, j\} \quad (13)$$

Looking for the equilibrium computationally is a simple process.

The expected utility terms in expression (13) are estimated each time the expression is calculated. For example, $\underline{x}_j \int_0^{\underline{x}_j}\left[u(x^{*i\underline{j}}; x^i)\right]\frac{1}{\underline{x}_j}dx^j$ must be estimated. First, to calculate $u(x^{*i\underline{j}}; x^i)$ for any given $x^j$ the Cabinet's best response is obtained computationally.[16] The expectation over $x^j$ is estimated by averaging over a number of values of $x^j$ chosen at small intervals across $[0, \underline{x}_j]$ using the trapezoidal rule.

Once a cut-off for MP$_i$, $\underline{y}_i$, satisfying (13) is found, the process is repeated for MP$_j$, setting $\underline{x}_i = \underline{y}_i$.[17] This procedure is iterated until the pair $(\underline{y}_i, \underline{y}_j)$ converges on $(\underline{x}_i + \epsilon_i, \underline{x}_j + \epsilon_j)$ where $\epsilon_k$, $k \in \{i, j\}$ is an error term, i.e., the process approximates the cut-off strategies.[18] The error terms can be made arbitrarily small by using a finer grid in estimating the expected utilities and in searching for solutions to (13). The equilibrium cut-off can be computed for any parameter values of the game as long as there exists $x \in [0, 1]$ satisfying $p_k - \beta(x^k - x)^2 - 2\alpha > 0$, $k \in \{i, j\}$. If the condition fails, the Cabinet's best response is no longer unique.

The equilibrium of the game is estimated for a number of parameter values. The aim is to generate predictions about how equilibrium strategies, and consequently levels of dissent, vary with the MPs' electoral strength and the cost of dissent. First I consider the MPs' electoral strength. In order to isolate the effects of change in electoral strength I hold other parameters of the model $(\alpha, \beta)$ fixed and vary MP$_j$'s prior re-election probability. The exercise is then repeated for different values of $\alpha$ and $\beta$ to check the robustness of the findings.

16 The Cabinet's best response was estimated using the `optimize` algorithm in `R`, which locates the maximum of $r_C$ given the MPs' dissent decisions.

17 The process begins with specifying an initial value for one of the cut-off strategies, say $\underline{y}_j$. Next, I look for a MP with ideal point $\underline{y}_i$, using the cut-off strategy $\underline{y}_i$, such that (13) holds for $k = i$. To find $\underline{y}_i$ I start by setting $\underline{y}_i = 0$ and calculate (13). If the RHS of (13), the policy benefit of dissent, is greater than the cost of dissent it implies that a MP of type $\underline{y}_i$ prefers dissent to acquiescence. Then $\underline{y}_i$ is increased by $10^{-1}$ and (13) is recalculated. This process continues until the gross benefit of dissent is lower than the cost of dissent at, say, $\underline{y}_i^1$. The same process is then repeat starting at $\underline{y}_i^1 - .1$ in increments of $10^{-2}$. This process is repeated until an estimate of $\underline{y}_i^8$ is obtained with an accuracy of approximately $10^{-8}$.

18 The number of iterations needed is generally between 15 and 20.

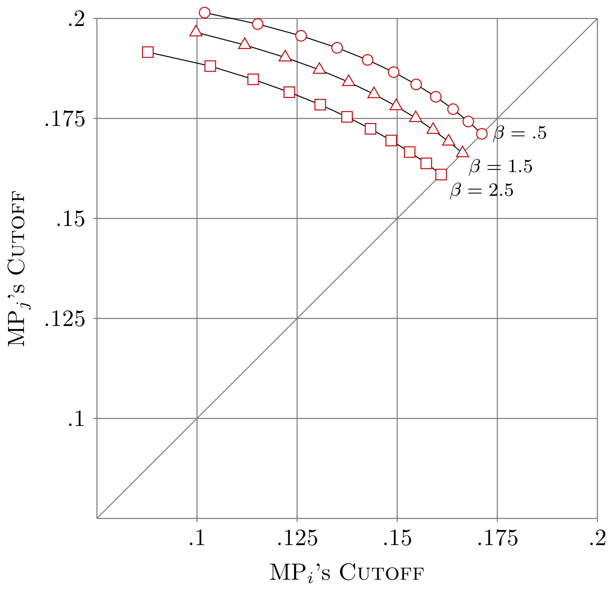


**Fig. 2** Equilibrium strategies and electoral volatility. Varying $p_j$ from 0.5 to 1, $p_i = 1$

The results that follow generally set one of the MP's prior re-election probabilities equal to one in order to maximize the range of parameter values that satisfy the conditions for the Cabinet's best response being unique. Cases where the MPs' prior re-election probabilities are not equal to one have also been examined and the results are substantively the same.[19] Examination of the first order conditions to the Cabinet's optimization problem, e.g., Eq. (5), also suggest that the restriction will generally have a modest impact if the MPs' prior re-election probabilities are not too far apart. This is also bourne out by a simulation below that compares the effect of the parties' electoral strength on dissent.

## 3.2 Comparative statics: equilibrium strategies

The effect of the MPs' prior re-election probabilities is of primary interests because it is widely discussed in the empirical literature. It also informs us about how the decision to dissent is influenced by the MPs' personal following and individual characteristics such as charisma, campaigning skills, resources, and other factors not related to policy platform of the MPs' party. Figure 2 plots the equilibrium cut-off strategies for MP$_i$ and MP$_j$ when $p_i = 1$ and $p_j$ is varied from 0.5 to 1. Each point represents the (lower) cut-off strategy for both MPs assuming a particular parameterization—MP$_i$'s strategy can be read of the horizontal axis while MP$_j$'s strategy can be read of the vertical axis. Each series, (□, △, ○), represents the candidates' equilibrium cut-off strategies

[19] Graphs considering such cases can be found at http://www.politicaldata.org/indridason.

for a particular parameterization of $\alpha$ and $\beta$ as $p_j$ is varied. The first thing to note about Fig. 2 is that the MP's cut-off strategies depend on the value of $p_j$. Consider the series plotted using a circle (•). The point closest to the 45° line represent the cut-off strategies when $p_j = p_i = 1$. For these parameter values MPs whose ideal points lie in the interval $[0.171, 0.829]$ do not dissent. The next point (•) represents the cut-off when $p_j = 0.95$, and so on in intervals of 0.05. Hence, the greater the disparity between the two MPs' prior re-election probabilities, the further away the point is from the 45° line. The intuition is clear; the greater the difference between the MPs' prior re-election probabilities, the more the Cabinet will favor the weaker MP and, hence, the stronger MP benefits less from dissent. Consequently, $MP_j$ becomes more likely to dissent (in the sense that less extreme types dissent) while $MP_i$ becomes less likely to dissent.

The second thing to note is the effect of the MPs' electoral volatility, $\beta$. As electoral volatility increases, Fig. 2 indicates that the MPs are less likely to dissent. This finding is somewhat counterintuitive because one might expect the MPs to be more likely to dissent if they pay a heavy price for policy outcomes that are unfavorable to their constituents. Equation (13), however, makes clear why this is the case. In equilibrium the cost of dissent must equal the MP's expected benefit of dissent. As electoral volatility the expected benefit of dissent increases. On average, given the strict concavity of the utility functions, the MP's expected benefit is higher when the other MP dissents than when he acquiesces. Thus, for the equilibrium condition in (13) to hold, acquiescence must occur with a higher probability. That is, in equilibrium, the candidates will adopt lower cut-off policies and dissent less frequently.

Proposition 1 showed existence of a symmetric cut-off point equilibrium but left the question of uniqueness open. While the type of analysis undertaken here is insufficient for establishing uniqueness, the computational results suggests that the equilibrium is unique. It is possible that the equilibria shown in Fig. 2 simply represent one of the equilibria that exist for the given parameter values. To investigate this possibility a number of starting values for the equilibrium search for each combination of parameter values were tried.[20] Naturally, the search for multiple equilibria is not exhaustive but the fact that not a single instance of multiple equilibria was revealed is suggestive of a more general property of the game.

Now consider the impact of the cost of dissent on the MPs' strategies. Figure 3 displays the results for different values of $\alpha$, varying $p_j$ while holding $p_i = 1$. While it is predictable that an increase in the cost of dissent leads to less dissent it is interesting to consider how the cost of dissent interacts with the MPs' electoral strength. As the gap between the MPs' electoral strength increases the more pronounced becomes the change in the electorally stronger MP's strategy. Hence, while a higher cost of dissenting is likely to discourage both MPs from dissenting, the electorally stronger MP reacts more sharply to the higher cost.

Together the figures above give a good indication of how the *individual* MPs' strategies are influenced by their prior probabilities of re-election, the cost of dissent,

[20] For each combination of parameter values the starting values for $MP_j$'s strategy were varied from 0 to 0.5 in intervals of 0.05.

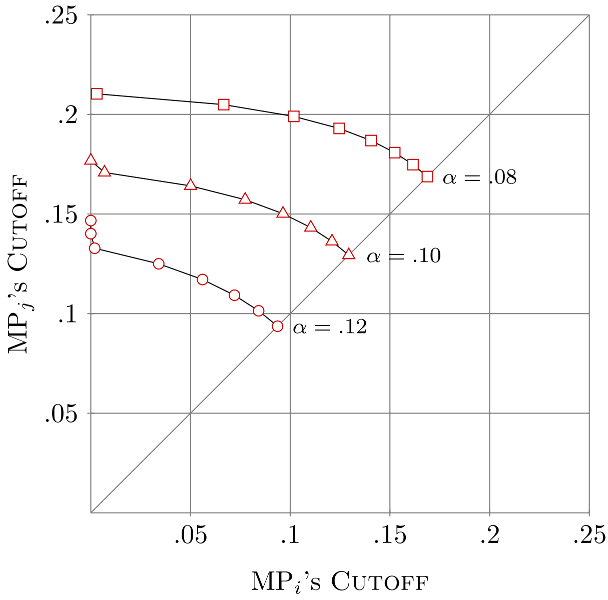


**Fig. 3** Equilibrium strategies and cost of dissent. Varying $p_j$ from 0.5 to 1, $p_i = 1$

and electoral volatility. Another variables of interest are the frequency and the levels of dissent (i.e., the expected number of dissenting MPs).[21]

## 3.3 Comparative statics: levels and frequency of dissent

Considering the level and frequency of dissent requires looking at the MPs' strategies simultaneously. Three outcomes are possible; no dissent, one MP dissents, or both MPs dissent. If $q_k$ is the probability of dissent associated with MP$_k$'s strategy $\underline{x}_k$[22] then the probability of no dissent equals $(1 - q_k)(1 - q_{\sim k})$, the probability of one MP dissenting equals $q_k(1 - q_{\sim k}) + (1 - q_k)q_{\sim k}$ and, finally, the probability of both MPs dissenting equals $q_k q_{\sim k}$. Here I consider the probability of both MPs dissenting as well as the probability of observing *some* dissent. The probability of observing some dissent equals the sum of the probability of one MP dissenting and the probability of both MPs dissenting—or simply one minus the probability of no dissent. These are the two outcomes that are most easily operationalized for empirical research.[23] The former, both MPs dissent, corresponds to a higher level of dissent or the average number of dissenting votes. The latter, some dissent occurs, corresponds to the probability of observing dissent on a given vote.

[21] Some, e.g., Kam (2002), use the term "depth of dissent" instead of level of dissent. The aggregate level of dissent is also likely to depend on the size of the majority that the Cabinet enjoys in the parliament.

[22] It should be obvious that $q_k = 2\underline{x}_k$.

[23] In contrast, the probability of a single MP dissenting is not easily operationalized—especially if the size of the Cabinet's majority does play a role in explaining the level of dissent.

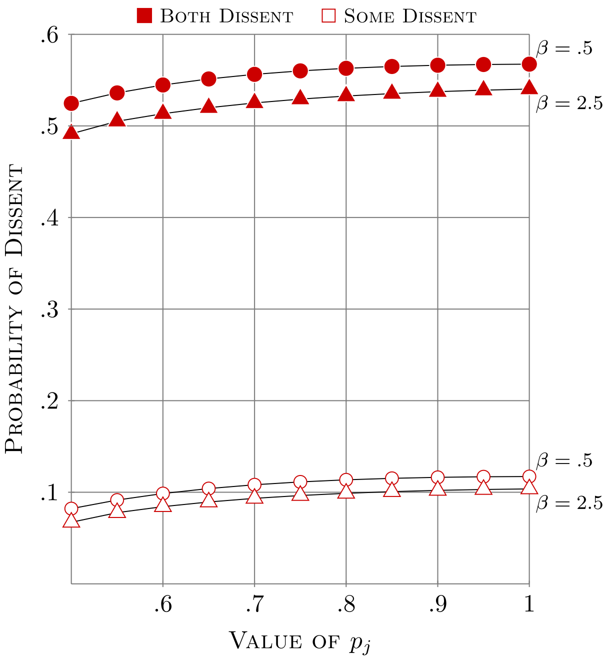


**Fig. 4** Probability of dissent and electoral volatility. Varying $p_j$ from 0.5 to 1, $p_i = 1$ $\alpha = 0.08$

Figures 4 and 5 plot the probability of dissent occurring against the prior re-election probability of MP$_j$ for two values of $\alpha$ and $\beta$. Both figures show that the probability of observing either type of dissent increases as the MPs' re-election probabilities approach parity. The effects are, however, fairly small unless there is a substantial gap between the MPs' re-election probabilities.

Figure 4 shows how electoral volatility influences the level of dissent. The counterintuitive effect of electoral volatility on dissent discussed above, is apparent in the figure. As electoral volatility increases, dissent becomes less likely. The effect is, however, fairly small—a fivefold increase in $\beta$ (from 0.5 to 2.5) only increases the probability of observing some dissent by four percentage points. The effect of the cost of dissent is much sharper as can be seen in Fig. 5. For example, a decrease in the cost of dissent from 0.1 to 0.08 increases the likelihood of observing dissent by nearly 10 percentage points. Thus, as the weaker MP's prior re-election probability ($p_j$) falls, the weaker MP has a stronger incentive to dissent but, at the same time, the stronger MP has even less incentive to dissent that outweighs MP$_j$'s increased willingness to dissent. The non-monotonic effect of $p_j$ on the probability of some dissent when $\alpha = 0.10$ and $\alpha = 0.12$ stems from the fact that MP$_i$'s cut-off strategy equals zero when $p_j$ is sufficiently low as can be seen in Fig. 3. That is, MP$_j$'s incentive to dissent continues to increase but MP$_i$ cannot dissent less.

The relationship between the party's electoral strength and dissent can also be considered. Intuitively, it appears plausible that the MPs' decisions to dissent depend not only on their individual chances of re-election but also on their party's overall

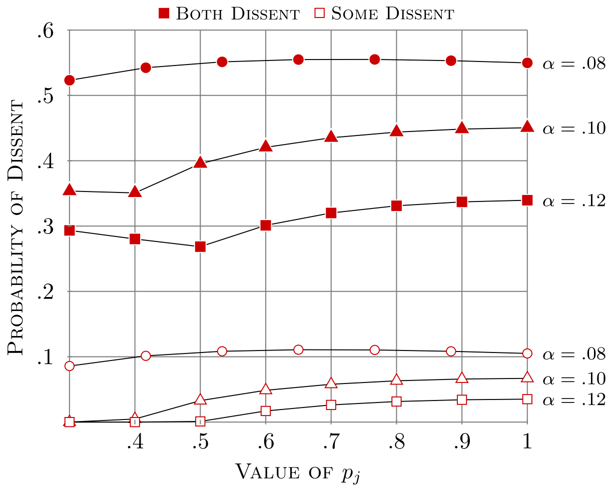


**Fig. 5** Probability of dissent and cost of dissent. Varying $p_j$ from 0.3 to 1, $p_i = 1$, $\beta = 1$

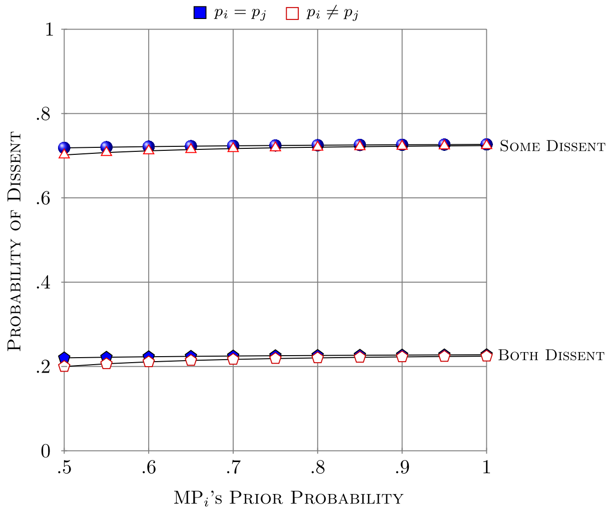


**Fig. 6** Probability of dissent: strong vs. weak parties. Varying $p_j$ from 0.5 to 1, $p_i = p_j$ and $p_i = p_j - 0.2$ $\alpha = 0.05$ and $\beta = 1$

electoral strength. The direction of the relationship is, however, not obvious. Intuitively, the MPs may have a greater incentive to dissent if their party is weak because they want to distance themselves from their party. On the other hand, it can be argued that MPs of stronger parties risk less by dissenting and, therefore, should be more prone to dissent. To consider this question I define an electorally weak party as a party whose members have low re-election probabilities. Varying the prior re-election probabilities of *both* MPs then provides information about the expected change in dissent. Figure 6 graphs the electoral strength of the party against the probability of dissent. The graph depicts the MPs' cut-off strategies when the two MPs' prior re-election probabilities are equal as well as unequal, and also considers different values of $\alpha$ and $\beta$. The results suggest a very weak relationship between the party's electoral strength and its MPs' incentive to dissent. The MPs are only slightly more likely to dissent as the party becomes stronger. The effect of party strength grows slightly stronger as the MP's prior re-election probabilities diverge.

Considering the effect of the party's electoral strength does reveal an additional insight into the expected pattern of dissent. As one can gather from Figs. 2 and 3 the cut-off strategies adopted by the two MPs when their prior re-election probabilities are equal are not symmetric, i.e., one of the MPs, a priori, is more likely to dissent than the other. As the party grows stronger the MPs' strategies become increasingly asymmetric, albeit very slightly. This implies that we should see more consistent patterns of dissent among MPs of electorally strong parties. That is, the pool of dissenting MPs is smaller in strong parties than in weak parties as the asymmetry of the MPs' strategies implies that some of them will dissent more frequently, and others less frequently, as party strength increases.

### 3.4 Comparative statics: policy outcome

I have argued, that costly dissent allows MPs to credibly transmit information about their policy preferences to the Cabinet. It is, therefore, interesting to consider the policy and electoral consequences of dissent. In order to do so, I simulate the policy outcome under different values of cost of dissent and electoral volatility for a randomly drawn sample of the MPs' ideal policies. A useful benchmark against which the policy outcome can be compared is the policy that maximizes the probability of both MPs being re-elected when the Cabinet faces no uncertainty about the MPs' preferences (henceforth I simply term this the "optimal policy"). To consider the policy effects of dissent, I calculate the average policy distance between the policy outcome under dissent and the optimal benchmark policy across the randomly drawn sample of MPs for each set of parameter values. Similarly, the policy distance between the optimal benchmark policy and the policy outcome when dissent is uninformative (or is, for some reason, not an option).[24] To consider the electoral consequences of dissent, I calculate the probability of both MP's being re-elected for the three cases, i.e., the optimal benchmark policy, the policy outcome under informative dissent, and the policy outcome under uninformative dissent.[25]

24 I use the term "uninformative dissent" here to refer to situations in which the Cabinet cannot update its beliefs on the basis of the MPs' decision to dissent, e.g., if dissent is costless.

25 Each simulation consisted of 1,000 independent draws of a pair of the MPs' ideal policies from a uniform distribution with the support [0, 1]. For each quadruple of parameter values $\{\alpha, \beta, p_i, p_j\}$ considered, the equilibrium cut-off strategies were computed using the process detailed in the previous section. The optimal benchmark policy was obtained using the `optimize` function in `R`. The policy outcome when dissent is uninformative equals 0.5. The equilibrium cut-off strategies determine which MPs dissent for each pair of MPs' ideal policies and the Cabinet's optimal policy can be obtained. Finally, the average policy distances and re-election probabilities are calculated by averaging across the 1,000 simulations.

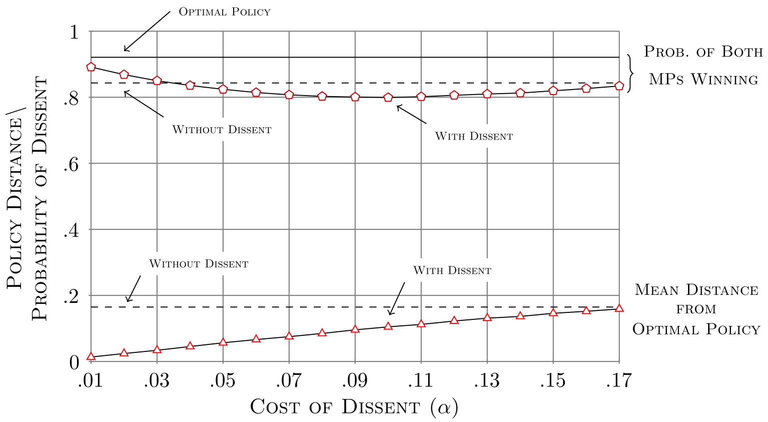


**Fig. 7** Cost of dissent: policy and reelection effects varying $\alpha$ from 0.01 to 0.17 $p_i = p_j = 1$ and $\beta = 1$

Figures 7 and 8 show the results of the simulations when the MPs' prior re-election probabilities are equal and as, respectively, the cost of dissent and electoral volatility is varied. The cost of dissent has a predictable effect on the mean policy distance from the optimal policy.[26] As cost of dissent tends towards zero, the MPs dissent ever more frequently and the mean policy outcome approached the optimal policy. At the other extreme, when the cost of dissent approaches being prohibitively expensive the mean policy outcomes tends towards the policy outcome when dissent is uninformative (or not an option). Focusing on the probability of both MPs being re-elected offers an insight into whether the Cabinet fares better or worse when the MPs have the ability to dissent to signal their preferences. By definition, dissent can never lead to a better outcome than simply knowing the MP's preferences with certainty beforehand but if the cost of dissent is sufficiently low then informative dissent improves on the situation where dissent is uninformative.[27]

Electoral volatility has a very limited effect on the mean distance between the optimal policy and the policy outcome under dissent.[28] This reflects the fact that change in electoral volatility has a very weak effect on the MPs' cut-off strategies as was seen in Fig. 2. As before, the average policy distance compares favorable with the policy outcome when dissent is uninformative. Turning to the probability of both MPs being elected, a higher degree of electoral volatility naturally has a negative effect on the probability. However, as electoral volatility increases, the probability under informative dissent becomes greater than the probability under uninformative dissent. The reason is that initially, when electoral volatility is low, the cost of dissent swamps

[26] Note that the optimal policy and the policy outcome when dissent is uninformative do not vary with the cost of dissent.

[27] Arguably the Cabinet's position is improved only for a small range of the values considered. Note, however, that dissent becomes beneficial around $\alpha = 0.03$, i.e. when a single dissenting vote decreases both MPs' re-election probabilities by a substantial amount (3% points).

[28] There is a very slight, barely perceptible in Fig. 8, increase in policy distance as electoral volatility increases.

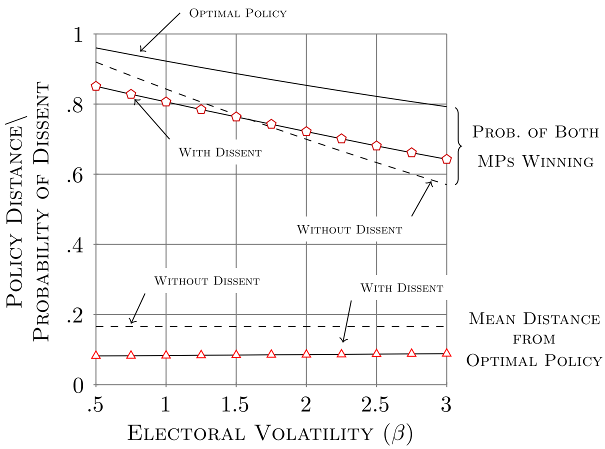


**Fig. 8** Electoral volatility: policy and reelection effects varying $\beta$ from 0.5 to 3 $p_i = p_j = 1$ and $\alpha = 0.08$

any benefits stemming from the choice of policy but as electoral volatility increases the distance between the Cabinet's implemented policy and the MPs' preferred policies weighs more heavily in the MPs' utility functions. Compared with the optimal policy, the policy outcome under informative dissent fares slightly worse as electoral volatility increases (as the MPs are slightly less likely to dissent).

In sum, Figs. 7 and 8 suggest that the Cabinet will be better off if the cost of dissent is sufficiently low and the electoral volatility is sufficiently high. The mean policy distance, on the other hand, is primarily affected by a decrease in the cost of dissent.

## 4 Conclusions

The fortunes of cabinets and their MPs are tied together. The cabinet must retain the confidence of the majority of the members of parliament to maintain control of government and to do so, the Cabinet must implement policies that are favorable to their MPs, i.e., policies that are likely to win them re-election. The problem facing the Cabinet is the heterogeneity of its MPs' preferences—each MP would like the Cabinet to implement a policy that corresponds to the preferences of his constituency's median voter. Knowing that the Cabinet must make a compromise that takes the MPs' heterogenous preferences into account, each MP has an incentive to misrepresent his preferences. This incentive reduces the MPs' messages to cheap talk. Costly action, i.e., dissent, helps resolve this problem. When dissent imposes a cost on the MP, the Cabinet can make inferences about the MP's preferences from the MP's actions. Thus, costly dissent opens up the possibility that the Cabinet can implement policies that are more likely to retain the Cabinet's majority in parliament. Although the Cabinet's ability to influence the cost of dissent has not been modeled here, this suggests that it may be rational for political parties to allow a certain degree of dissent within their ranks.

**Table 1** Summary of comparative statics

| | Individual dissent | Frequency of dissent | Level of dissent |
|---|---|---|---|
| Cost of dissent | – | – | – |
| Prior probability of re-election | – | +[†] | +[†] |
| MP's electoral volatility | – | – | – |
| Party's electoral strength | +[††] | +[††] | +[††] |

[†] Refers to the difference between the MPs' prior probability of re-election
[††] The marginal effect is very small

Table 1 summarizes the comparative statics suggested by the computational results. The probability of dissent is negatively correlated with the cost of dissent. A MP located in an extreme constituency can expect to obtain a more favorable policy outcome by dissenting than a MP who represents a moderate constituency. However, the magnitude of the Cabinet's concession depends on how extreme the MP's constituency is and, as the cost of dissent rises, less extreme MPs will no longer find it beneficial to dissent. As the cost of dissent applies equally to all MPs, dissent should be observed less frequently and, when it occurs, fewer MPs are likely to dissent. That is, the MPs' equilibrium cut-off strategies move away from mid-point of the policy dimension.

The likelihood of a MP dissenting is decreasing in the MP's prior re-election probability as the Cabinet's optimal policy benefits the weaker MP. However, a change in a MP's re-election probability also influence the other MPs' incentives to dissent. As the weaker MP's re-election probability increases, his incentive to dissent decreases but these are countered by a greater incentive for the stronger MP's to dissent. On the whole, the latter outweighs the former and the likelihood of observing dissent increases as the re-election probabilities become more similar. The probability of both MPs dissenting, and the expected number of dissenting MPs, increases as well. This result is intuitively clear. When the re-election probabilities of the MPs are unequal, the secure MP has little to gain from dissenting. By dissenting, he will at best marginally improve his chances of re-election. As the re-election probabilities become more equal both MPs face non-negligible incentives to dissent, and in the aggregate the level of dissent exceeds the level that occur when one MP is marginal and the other secure.

One might expect electoral volatility to increase dissent. Interestingly, however, the effects of electoral volatility on the MPs' equilibrium behavior is the exact opposite. Intuitively the reason is that as electoral volatility increases both MPs would appear to have an added incentive to dissent. However, if a MP chooses to dissent with a high probability, the other MP stands to suffer an unfavorable outcome more frequently and is, therefore, induced to dissent more frequently as well. Instead an equilibrium occurs where the MPs temper each other incentives to dissent by dissenting infrequently themselves. As electoral volatility increases, each MPs' incentive to dissent increases and to reduce that incentive for each other the MPs must choose lower levels of dissent. It is apparent that for a sufficiently high level of electoral volatility such equilibria will not exist. However, the above result holds for the space of parameters that satisfy the conditions for the Cabinet having a unique best response.

The model allows consideration of whether electorally weak and strong parties should experience different levels of dissent. The differences in the probability and the level of dissent exhibit only a very slight upward trend as party strength increases. Cabinet parties should, therefore, not be expected to experience substantial changes in levels of dissent as their popularity waxes and wanes.

Finally, the policy and electoral consequences of dissent have been simulated by considering samples of MP's with randomly drawn ideal policies. Compared against the policy that the Cabinet would choose to implement if it knew the MPs' preferences with certainty, the policy outcome under dissent outperforms the policy outcome when dissent is uninformative or is "outlawed". As the cost of dissent tends towards zero, the policy outcome under dissent approaches the optimal policy but electoral volatility has almost no impact on the mean policy distance from the optimal policy.

Would cabinets ever prefer costly dissent to quiet and obedient legislature? In terms of the electoral consequences of dissent the answer is that under certain circumstances cabinets will benefit from MPs being able to exercise costly dissent. First, not surprisingly, if the cost of dissent is relatively low, the cabinet can obtain some information about its MPs' preferences and implement a policy that increases its chances of retaining the majority. Second, the value of information about the MPs' preferences increases with electoral volatility and eventually outweighs the cost of their dissent.

The conditions for dissent having beneficial effects for the Cabinet may appear somewhat restrictive. However, as noted above, dissent is beneficial if a single dissenting vote decreases each MPs' re-election probability by three percentage points or less (simulation in Fig. 7). Most MPs would regard this as a substantial cost. More importantly, in this paper I have assumed that the cost of dissent takes a particular form, i.e., that dissent hurts the party's reputation and, thus, reduces every MPs' chance of re-election. In reality, a single vote of dissent is unlikely to have such large effects on the party's reputation. Persistent dissent and large rebellions are more likely to hurt the party's reputation. MPs may also face a variety of other costs, some of which can be manipulated by the party leadership, e.g., demotions from parliamentary or party positions.[29]

The findings presented here also offers some advice as to how to model legislators' dissent empirically. Cowley (2002), for example, considers the effect of the MP's electoral strength on the probability of dissent, which turns out to be "statistically insignificant". The model presented here suggests that Cowley's model is mis-specified, i.e., it was shown above that the effect of the MP's electoral strength on his willingness to dissent depends on how extreme his preferences are and the empirical model should, therefore, include an interaction between electoral strength and preferences.[30]

[29] de Dios (1999) reports that Spanish parliamentary parties go as far as imposing fines on MPs that vote against the party-line. As it is the nature, and not the type, of costly dissent that permits informative transmission of information, this suggests that the Cabinet may strategically manipulate the cost of dissent as to maximize its likelihood to stay in power.

[30] In addition, Cowley (2002) includes two variables that measure electoral strength (an indicator for unexpected victors and percentage majority) thus introducing collinearity into the model, which potentially masks a significant relationship. However, Cowley's analysis does neither report either estimated standard error nor $t$-statistics and is not complete enough to draw accurate inference about the role of preferences or electoral strength.

## Appendix A

**Lemma 1** *If* $f(x)$ *is a symmetric density function about* 0.5 *then* $r_c(x) = \Big[p_i + \int_{\underline{x}_i}^{\overline{x}_i} u(x; x^i) f(x^i) dx^i\Big] * \Big[p_j + \int_{\underline{x}_j}^{\overline{x}_j} u(x; x^j) f(x^j) dx^j\Big]$ *is maximized at* $x = 0.5$.

*Proof* Note that:

$$\int_{\underline{x}_k}^{\overline{x}_k} \frac{dw(x^k - x)}{dx}\Big|_{x=0.5} f(x^k) dx^k = \int_{\underline{x}_k}^{0.5} \frac{dw(x^k - x)}{dx}\Big|_{x=0.5} f(x^k) dx^k + \int_{0.5}^{\overline{x}_k} \frac{dw(x^k - x)}{dx}\Big|_{x=0.5} f(x^k) dx^k \tag{14}$$

By symmetry of $u(x; x_k)$:

$$\frac{dw(x^k - x)}{dx}\Big|_{x=0.5} = -\frac{dw(x - x^k)}{dx}\Big|_{x=0.5} \tag{15}$$

Substituting (15) into (14) the RHS becomes:

$$\int_{\underline{x}_k}^{0.5} \frac{dw(x^k - x)}{dx} f(x^k) dx^k\Big|_{x=0.5} - \int_{0.5}^{1-\underline{x}_k} \frac{dw(x^k - x)}{dx} f(x^k) dx^k\Big|_{x=0.5} \tag{16}$$

By symmetry of $f(x^k)$ it is known that:

$$\int_{a}^{b} f(x^k) dx^k = \int_{1-b}^{1-a} f(x^k) dx^k \tag{17}$$

and finally, by substituting (17) into (16):

$$\begin{aligned} &\int_{\underline{x}_k}^{0.5} \frac{dw(x^k - x)}{dx} f(x^k) dx^k\Big|_{x=0.5} - \int_{1-\underline{x}_k}^{0.5} \frac{dw(x - (1 - x^k))}{dx} f(1 - x^k) dx^k\Big|_{x=0.5} \\ &= \int_{\underline{x}_k}^{0.5} \frac{dw(x^k - x)}{dx} f(x^k) dx^k\Big|_{x=0.5} - \int_{\underline{x}_k}^{0.5} \frac{dw(x^k - x))}{dx} f(x^k) dx^k\Big|_{x=0.5} = 0 \end{aligned} \tag{18}$$

This can be rewritten as:

$$\left.\frac{dE[u(x;x^k)|d_k=0]}{dx}\right|_{x=0.5}=0 \tag{19}$$

Hence, by (19) the Cabinet's optimal policy when neither MP dissents is at $x^*=0.5$ as in expectations both of the MPs' re-election probabilities are maximized at $x^*$. □

**Lemma 2** *Without loss of generality assume* $x^i<x^j$. *The Cabinet's response to a change in* $x^i$, *when* $MP_i$ *dissents, is positive but less than unity, i.e.,* $0\le\frac{\partial x^*}{\partial x^i}\le 1$.

*Proof* Now assume that $\exists x^i,\tilde{x}^i$, such that $\tilde{x}^i=x^1+\epsilon$, $\epsilon>0$, but small, $x^i\notin[\underline{x}_i,\overline{x}_i]$, and $\tilde{x}^i\notin[\underline{x}_i,\overline{x}_i]$. Now suppose to the contrary that $\frac{\partial x^*}{\partial x^i}<0$, which implies that $\exists\epsilon$ such that $x^*>\tilde{x}^*$ where $\tilde{x}^*$ is the Cabinet's optimal policy when MP$_i$'s type is $\tilde{x}^i$. By (7) it is known that $\tilde{x}^*$ solves

$$\frac{p_i+u(\tilde{x}^*;\tilde{x}^i)-2\alpha}{p_j+u(\tilde{x}^i;x^j)-2\alpha}=-\frac{\left.\frac{\partial u(x;\tilde{x}^i)}{\partial x}\right|_{x=\tilde{x}^*}}{\left.\frac{\partial u(x;x^j)}{\partial x}\right|_{x=\tilde{x}^*}}.$$

For the cases where MP$_j$ does not dissent, replace the denominators with the appropriate expectations but the same logic applies. As $x^*>\tilde{x}^*$ it follows that $r^i(x^*)<r^i(\tilde{x}^*)$ and that

$$-\frac{\left.\frac{\partial u(x;\tilde{x}^i)}{\partial x}\right|_{x=\tilde{x}^*}}{\left.\frac{\partial u(x;x^j)}{\partial x}\right|_{x=\tilde{x}^*}}<-\frac{\left.\frac{\partial u(x;\tilde{x}^i)}{\partial x}\right|_{x=x^*}}{\left.\frac{\partial u(x;x^j)}{\partial x}\right|_{x=x^*}},$$

thus contradicting

$$\frac{r^i(x^*)}{r^j(x^*)}=-\frac{\left.\frac{\partial u(x;x^i)}{\partial x}\right|_{x=x^*}}{\left.\frac{\partial u(x;x^j)}{\partial x}\right|_{x=x^*}}.$$

To show that the marginal change in the cabinets policy is less than unity it suffices to point out that to argue that $\frac{\partial x^*}{\partial x^i}\le 1$ is equivalent to arguing that $\frac{\partial x^*}{\partial x^j}\le 0$ since only the distance between the (expected) ideal points of the MPs and not their location determines the government's payoff. The argument for $\frac{\partial x^*}{\partial x^j}\le 0$, however, is symmetric to the argument above. □

**Lemma 3** *The functions* $u(x^{*ij};x^k)$, $u(x^{*k};x^k)$, *and* $u(x^{*\sim k};x^k)$, $k\in\{i,j\}$, *are concave in* $x^k$.

*Proof* Start with $u(x^{*ij};x^i)$. Without loss of generality assume that $x^i<x^j$. Lemma 2 shows that $0\le\frac{\partial x^*}{\partial x^i}\le 1$, and by Corollary 1 we have $-1\le\frac{\partial x^*}{\partial x^j}\le 0$. For all $x^i,\hat{x}^i\in[0,1]$ such that $x^i<\hat{x}^i$ and $x^i_\theta=\theta x^i+(1-\theta)\hat{x}^i$, Lemma 2 implies that $x^{*ij} \leq x_\theta^{*ij} \leq \hat{x}^{*ij}$ and $x^{*ij} - x^i \geq x_\theta^{*ij} - x_\theta^i \geq \hat{x}^{*ij} - \hat{x}^i, \forall\theta \in (0, 1)$. That is, the Cabinet's policy choice for any convex combination of $x^i$ and $\hat{x}^i$ has to lie between the policies the Cabinet would choose were MP$_i$'s ideal point at that location. Since $u(x^{*ij}; x^i)$ is a function of the distance between $x^{*ij}$ and $x^i$, and $\frac{\partial u(x^{*ij};x^i)}{\partial x}$ is a function of the difference between $x^{*ij}$ and $x^i$, it must be that $\frac{\partial u(x^{*ij};x^i)}{\partial x^i} \geq \frac{\partial u(x_\theta^{*ij};x_\theta^i)}{\partial x^i} \geq \frac{\partial u(\hat{x}^{*ij};\hat{x}^i)}{\partial x}$ and consequently $\frac{\partial^2 u(x^{*ij};x^i)}{\partial x^2} \leq 0$. The proof for each of the other functions follows the same argument. □

**Lemma 4** *The terms* $u_i(x^{*i}) - u_i(0.5)$ *and* $\underline{x}_j \left[\int_0^{\underline{x}_j} \left[u_i(x^{*i\underline{j}}) - u_i(x^{*\underline{j}})\right] \frac{1}{\underline{x}_j} dx^j\right] + (1 - \underline{x}_j) \left[\int_{\overline{x}_j}^1 \left[u_i(x^{*i\overline{j}}) - u_i(x^{*\overline{j}})\right] \frac{1}{1-\overline{x}_j} dx^j\right]$ *are strictly decreasing in* $x^i$ *in the interval* $[0, 0.5)$*, and strictly increasing in* $x^i$ *in the interval* $(0.5, 1]$.

*Proof* Start with the term $u_i(x^{*i}) - u_i(0.5)$. First note that both the functions are increasing in $x^i$ in the interval $[0, 0.5)$ and are at their maxima at $x^i = 0.5$. Taking the first derivative with respect to $x^i$ yields:

$$\frac{\partial u_i(x^{*i})}{\partial x^{*i}} \frac{dx^{*i}}{dx^i} + \frac{\partial u_i(x^{*i})}{\partial x^i} - \left[ \left.\frac{\partial u_i(x^{*i})}{\partial x^{*i}}\right|_{x^*=0.5} \frac{dx^{*i}}{dx^i} + \left.\frac{\partial u_i(x^{*i})}{\partial x^i}\right|_{x^*=0.5} \right] \tag{20}$$

The first term in the bracket equals zero as $\frac{dx^{*i}}{dx^i} = 0$ when MP$_i$ does not dissent. By the concavity of $u_i(\bullet)$, $\frac{\partial u_i(x^{*i})}{\partial x^i} < \left.\frac{\partial u_i(x^{*i})}{\partial x^i}\right|_{x^*=0.5}$. It is also known that when $x^i \in [0, 0.5)$, $\frac{\partial u_i(x^{*i})}{\partial x^{*i}} < 0$, and $\frac{dx^{*i}}{dx^i} \geq 0$. Hence it must be the case that (20) is strictly negative in $[0, 0.5)$. To show that the second term is decreasing in $x^i$ in the interval $[0, 0.5)$, fix $x^{\underline{j}} \in [0, \underline{x}^j]$, and let $x^{\overline{j}} = \overline{x}^j + x^{\underline{j}}$. If it is shown that $u_i(x_i^{*i\underline{j}}) - u_i(x^{*\underline{j}}) + u_i(x^{*i\overline{j}}) - u_i(x^{*\overline{j}})$ is decreasing in $x^i$ in the interval $[0, 0.5)$ when the first two terms are evaluated at $x^{\underline{j}}$, and the second two at $x^{\overline{j}}$, the same has to hold as when integrating over the intervals $[0, \underline{x}^j]$ and $[\overline{x}^j, 1]$ in the statement of the lemma. Note that by supposition, $\overline{x}^j = 1 - \underline{x}^j$, so there is no need not worry about the weights that the MP$_i$ assigns to type of MP$_j$'s dissent. Differentiating $u_i(x^{*i\underline{j}}) - u_i(x^{*\underline{j}}) + u_i(x^{*i\overline{j}}) - u_i(x^{*\overline{j}})$ with respect to $x^i$ yields:

$$\begin{aligned} &\frac{\partial u_i(x^{*i\underline{j}})}{\partial x^{*i\underline{j}}} \frac{dx^{*i\underline{j}}}{dx^i} + \frac{\partial u_i(x^{*i\underline{j}})}{\partial x^i} - \frac{\partial u_i(x^{*\underline{j}})}{\partial x^i} \\ &\quad + \frac{\partial u_i(x^{*i\overline{j}})}{\partial x^{*i\overline{j}}} \frac{dx^{*i\overline{j}}}{dx^i} + \frac{\partial u_i(x^{*i\overline{j}})}{\partial x^i} - \frac{\partial u_i(x^{*\overline{j}})}{\partial x^i} \end{aligned} \tag{21}$$

Divide the interval $[0, 0.5)$ up into three intervals: (i) $[0, \hat{x}^i]$ where $\hat{x}^i = \arg\max_{x \in [0,0.5)} \int_0^{\underline{x}_j} u_i(x^{*i\underline{j}}) \frac{1}{\underline{x}_j} dx^j$, (ii) $(\hat{x}^i, \tilde{x}^i)$ where $\tilde{x}^i = \arg\max_{x^i \in [0,0.5)} \int_0^{\underline{x}_j} u(x^{*\underline{j}}; x^i) \frac{1}{\underline{x}_j} dx^j$, and (iii) $[\tilde{x}^i, 0.5)$. Intuitively, $\hat{x}^i$ and $\tilde{x}^i$ are the maxima of the functions $u_i(x^{*i\underline{j}})$ and $u_i(x^{*\underline{j}})$, respectively. Dividing the interval $[0, 0.5)$ into these three regions is convenient as the derivatives of the two functions can easily be signed in these intervals since the functions are concave. Now consider the three cases. In (i) $0 < \frac{\partial u_i(x^{*i\underline{j}})}{\partial x^i} < \frac{\partial u_i(x^{*\underline{j}})}{\partial x^i}$ since $x^{*i\underline{j}} \leq x^{*\underline{j}}$, and likewise $0 < \frac{\partial u_i(x^{*i\overline{j}})}{\partial x^i} < \frac{\partial u_i(x^{*\overline{j}})}{\partial x^i}$ since $x^{*i\overline{j}} \leq x^{*\overline{j}}$. Since the government optimal policy always lies between the MPs' ideal points when both dissent the two multiplicative terms in (21) must be less than or equal to zero. Therefore, (21) is negative in the interval $[0, \hat{x}^i]$. In (ii) $\frac{\partial u_i(x^{*i\underline{j}})}{\partial x^i} < 0$ and $\frac{\partial u_i(x^{*i\underline{j}})}{\partial x^{*i\underline{j}}} \frac{dx^{*i\underline{j}}}{dx^i} > 0$, but by Lemma 1 $\frac{dx^{*i\underline{j}}}{dx^i} \leq 1$ and $\frac{\partial u_i(x^{*i\underline{j}})}{\partial x^i} = -\frac{\partial u_i(x^{*i\underline{j}})}{\partial x^{*i\underline{j}}}$ so together the first three terms of (21) are less than zero. The last three terms of (21) are negative since $0 < \frac{\partial u_i(x^{*i\overline{j}})}{\partial x^i} < \frac{\partial u_i(x^{*\overline{j}})}{\partial x^i}$ since $x^{*i\overline{j}} \leq x^{*\overline{j}}$ as in (i). Hence, (21) is negative in the interval $[\hat{x}^i, \tilde{x}^i)$. (iii) Now $\frac{\partial u_i(x^{*i\underline{j}})}{\partial x^i} < 0$, and $\frac{\partial u_i(x^{*\underline{j}})}{\partial x^i} < 0$. By the concavity of the functions $u_i(\bullet)$: $\frac{\partial u_i(x^{*\underline{j}})}{\partial x^i} > -\frac{\partial u_i(x^{*\overline{j}})}{\partial x^i}$, $\frac{\partial u_i(x^{*i\underline{j}})}{\partial x^i} > -\frac{\partial u_i(x^{*i\overline{j}})}{\partial x^i}$, and $\frac{\partial u_i(x^{*i\underline{j}})}{\partial x^{*i\underline{j}}} \frac{dx^{*i\underline{j}}}{dx^i} < \frac{\partial u_i(x^{*i\overline{j}})}{\partial x^{*i\overline{j}}} \frac{dx^{*i\overline{j}}}{dx^i}$. Hence, (21) is negative in the interval $[\tilde{x}^i, 0.5)$ . The derivative in (21) is therefore negative in the interval $[0, 0.5)$. It remains to show that (21) is positive in the interval $(0.5, 1]$. By symmetry of the actors' preferences and the symmetry of the distribution the MPs are drawn from, the argument is symmetric. □

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