

## WHEN TO RUN AND WHEN TO HIDE: ELECTORAL COORDINATION AND EXIT

INDRIDI H. INDRIDASON\*

Elections represent a coordination problem for voters and candidates for office. Electoral coordination is also the causal mechanism behind any explanation of the relationship between electoral systems and the number of parties. I present a dynamic model of electoral coordination with candidate exit. The model extends two important results from the literature to a dynamic setting. The extension of Duverger's Law and the median-voter theorem also offers a simultaneous prediction of the number of parties and their ideological positions. Coordination failure is shown to be possible in a mixed-strategy equilibrium.

### 1. INTRODUCTION

THE LITERATURE exploring the relationship between electoral systems and party systems is extensive. The best known statement of this relationship is Duverger's Law (and Hypothesis). Duverger (1954) identified strategic voting as the reason why electoral competition in single-member districts under plurality rule leads to a two-party system. That is, electoral coordination is achieved by voters as they seek to avoid "wasting" their votes on candidates that have no chance of winning. This is also the case in most models of electoral competition (see, for example, Feddersen, 1992; Fey, 1997; Ledyard, 1984; Myerson and Weber, 1993; Palfrey, 1989). The general result is that the voters' strategic behavior leads to an equilibrium in which only a handful of candidates receive any votes. The exclusive focus on voters leaves much to be said about elections. Before the voters go to the polling booth, parties and coalitions form and disband, candidates enter and exit, campaign contributions are solicited and made, polls are taken and published, etc. Much of this activity is directed at determining which candidates, or parties, eventually appear on the ballot.

Duverger (1954) thus provided important insights into the effects of electoral systems on party systems but had little to say about the role of the political elite in achieving electoral coordination, though deciding to contest

\*Corresponding author: Indridi H. Indridason, Department of Politics & International Relations, University of Oxford, Manor Road Building, Oxford OX1 3UQ, UK. E-mail: [indridi.indridason@politics.ox.ac.uk](mailto:indridi.indridason@politics.ox.ac.uk)

an election and the choice of a platform are clearly strategic decisions.<sup>1</sup> The U.S. presidential primaries are a good example of why the process of electoral coordination, or candidate survival, matters. Typically several politicians indicate that they seek the nomination well in advance of the primary. Who the real contenders are becomes clearer as the primary draws closer and the politicians re-evaluate their candidacies in the light of their support and the success of their fund-raising efforts. The field of candidates is winnowed further as the results of the first primaries come in and usually all but one candidate have exited from the race before the last primary takes place.<sup>2</sup>

Neglecting the process of attrition is not without consequence. We are not only interested in the number of parties in equilibrium. Which candidates survive the process of electoral coordination to become contenders for office, and the policies they adopt, is generally of no less interest than the number of candidates.

In this article I consider a model of candidate exit where both the number of candidates and their identity are determined endogenously. As the candidates are assumed to be unable to make credible commitment, the model also yields predictions about the equilibrium policy platforms. The main result mirrors Duverger's Law in that the number of candidates is at most two. Moreover, the results are closely tied to the familiar median voter theorem. In the (pure strategy) equilibria of the game all but the median voters' most preferred candidates exit immediately except in knife-edge situations.

Candidate positioning, alongside the issue of the equilibrium number of parties, has long been a prominent topic in the study of electoral systems. Early on, Hotelling (1929) provided a model in which the candidates converge on the median voter's preferred policy positions. Numerous modifications of Hotelling's (1929) model have been considered. Candidate positioning has been examined, for example, when candidates are motivated by policy rather than winning office (Calvert, 1985; Wittman, 1977), the policy space is multidimensional (Plott, 1967), legislatures are elected (rather than a single office) (Austen-Smith, 1984; Austen-Smith and Banks, 1988), voters act strategically (Ledyard, 1984), and one candidate has valence advantage (Groseclose, 2001).<sup>3</sup>

However, models of candidate positioning have typically not concerned themselves with analyzing the equilibrium number of parties (and vice

<sup>1</sup>See, e.g., Besley and Coate (1997) and Osborne and Slivinski (1996) for models in which the candidates decide whether to run for office. Cox (1997) also argues that the political elite plays an important role in this process. On candidate positioning see, e.g., Downs (1967), Hotelling (1929), and, for a nice overview of the literature, Osborne (1995).

<sup>2</sup>The model presented here does not capture all of the aspects of the U.S. presidential primaries. For example, in the primaries the candidates accumulate electoral college votes that influence their (expected) relative standing in the primary.

<sup>3</sup>These are only a few examples; see Osborne (1995) for a good survey of the literature.

versa). There are a few notable exceptions. Osborne (2000) and Palfrey (1984), for example, examine entry deterrence in a model of candidate positioning. While their primary concern is not with the number of parties, the possibility of entry deterrence has clear implications for the number of parties that contest elections. The number of candidates and their policy positions are, however, determined endogenously in the citizen-candidate models of Besley and Coate (1997) and Osborne and Slivinski (1996). In their models, each voter can announce her candidacy but can only credibly commit to her ideal policy.<sup>4</sup>

There are numerous reasons why the role of the political elite in achieving electoral coordination should receive greater attention. The political elite is likely to have far greater stakes in the electoral outcome than the voters and is, thus, more likely to respond to the incentives provided by the electoral system. Even if that were not the case, the incentive for the voter to behave strategically should be vastly reduced by the fact that the likelihood that each vote influences the outcome of the elections is very small – strategic voting behavior is a puzzle in the same sense that voter turnout is a puzzle.<sup>5</sup> Empirical research has furthermore shown that levels of strategic voting are generally rather low (see, e.g., Alvarez and Nagler, 2000; Blais, 2002). Shifting the focus to the strategies adopted by the political elite is therefore likely to enhance our understanding of electoral coordination and, more generally, electoral politics.

Whereas the standard approach to modeling electoral coordination is to model it as a voting game, it is modeled here as a variant of the war of attrition (Maynard Smith, 1974). I examine a dynamic model of electoral coordination where the number of candidates and policy platforms are exogenously determined. There are two groups of actors, candidates and voters. The model consists of a (potentially infinite) sequence of polls or ballots. Before each poll the candidates decide whether they wish to stay in the race or to exit. If a candidate decides to run he incurs some (opportunity) cost. The two stages are repeated until some candidate garners the support of a majority of the voters.

The model differs in several ways from existing models in the literature on the war of attrition. The model is an  $N$ -player war of attrition in discrete time with complete information. The war of attrition is most commonly modeled in continuous time (see, e.g., Bulow and Klemperer, 1999; Fudenberg and Tirole, 1986) though there are exceptions to this (see, e.g., Hendricks and Wilson, 1985; Kapur, 1995). Majority of the literature has focused on two player wars of attrition although, again, there are some

<sup>4</sup> Feddersen (1992) builds on a similar idea. In his model there are no candidates but instead voters cast their votes for some policy.

<sup>5</sup> Voting strategically requires an understanding of how the electoral system works and knowledge of the candidates' platforms and their likelihood of winning. It seems reasonable to assume that acquiring this information is costly.

notable exceptions (see, e.g., Bulow and Klemperer, 1999; Kapur, 1995). The model analyzed here also assumes a certain asymmetry among the actors, i.e., the candidates are ranked according to majority preference relation.<sup>6</sup> The implication of this is that the order in which the candidates exit matters for who emerges as the winner.<sup>7</sup>

It is worth emphasizing some of the features of the model. First, and most importantly, it does not employ any mechanisms to force electoral coordination other than the electoral rule itself. Electoral coordination is thus endogenous to the model and can take place either at the elite level or at the voter level. Second, the model is dynamic, allowing for delayed coordination. Third, it does not assume that voters achieve coordination (though I consider strategic voting below).

The model offers a more realistic picture of electoral politics that gives us insights into issues such as party system characteristics, including both the number of parties and their policy platforms, the potential for coordination failure and the rate of electoral coordination. It is also important to note that the generality of the model lends it to various applications – not only does it have implications for elections but also for any process of selection leading up to an election, including formal or informal candidate and leadership selection processes.

The formal structure of the model mirrors the speakership election of the U.S. House of Representatives. The Speaker of the House is elected by a majority run-off that differs from the “traditional” dual ballot majority run-off in two ways. First, there are no hurdles that the candidates must pass in order to advance to the next round; any candidate, no matter how badly he fared on the previous ballot, can run again. Second, the number of ballots is not restricted to two, but is only limited by the number of rounds required to produce a majority winner. Thus the election could, technically, go on indefinitely.<sup>8</sup> In the early years of the U.S. Congress significant delays occurred occasionally in Speakership elections. Between 1820 and 1860 more than 10 ballots were taken on seven occasions before a Speaker was elected. On two occasions, in 1849 and 1855, the House “gave up” on the majority requirement and adopted plurality rule to elect the Speaker after 62 and 132

<sup>6</sup> Myatt (2005) considers an asymmetric war of attrition where the actors’ costs are drawn from different distributions.

<sup>7</sup> Supposed there are three candidates (1, 2, 3) that are linearly ranked by the majority preference relation in the order of their labels. Then Candidate 2 loses if Candidate 3 exits but wins if Candidate 1 exits.

<sup>8</sup> The electoral systems were also used to select presidential candidates in the United States before presidential primaries were adopted. A similar multi-ballot system, where the candidate with the fewest votes is eliminated, is used for the election of the leader of British Conservative Party. Interestingly, until 1965 the party had no formal mechanism for leadership selection. Instead the leader “emerged” from the “magic circle” of prominent Tory members (Quinn, 2005). Finally, papal elections are conducted under this electoral system with the exception that a two-thirds majority is required on the first 28 ballots. After the 28th ballot an absolute majority of the cardinals can lower the requirement to an absolute majority.

ballots (Stewart, 1999). During the election of 1855 the members of the House engaged in various strategic maneuvers (Jenkins and Nokken, 2000). Candidates exited and new entered, and voters switched their votes back and forth. But in this particular case it was all for nothing – the system in use proved to be irresolute and it was not until plurality rule was adopted that the election was brought to a close. While cases like these are perhaps unusual, the speakership elections exhibit behavior that models of electoral coordination should capture. The field of candidates is winnowed as Du-vergerian incentives to vote strategically kick in, and as candidates amass information relevant to the viability of their candidacy (see, for example, Feddersen, 1992; Ledyard, 1984; Palfrey, 1989). There may, however, be more to electoral coordination than the wasted vote phenomena and elite anticipation of it affecting entry and exit decisions.

Thus, electoral coordination does not always take place seamlessly. This begs the question of what conditions lead to coordination failures. For example, the patterns of electoral competition in newly established democracies exhibit similar patterns to the Speakership elections where new parties enter and existing parties merge, split, or exit with considerable frequency (see, for example, Kaminski, 2001; Montgomery, 1999; Moser, 2001). The greater stability in most established democracies suggests that the institutionalization of party systems plays an important role in preventing coordination failures. The French presidential election of 2002 showed, however, that party institutionalization is no guarantee against coordination failures.<sup>9</sup>

## 2. A MODEL OF ELECTORAL COORDINATION

There are two sets of actors, voters and candidates. Let  $N$  be a set of  $n$  voters with  $n$  odd. Voters care only about policy outcomes. The policy space,  $X \subset \mathbb{R}$ , is uni-dimensional. Voter  $i$ 's utility function is a mapping  $u_i: X \rightarrow \mathbb{R}$  and the voter's ideal point is at  $x^i$ , i.e.,  $x^i = \arg \max_{x \in X} u_i(x)$ . The profile of voters' preferences is assumed to be single-peaked on  $X$ , i.e., for all  $i \in N$ : (i) there exists  $x^i \in X$  such that  $u_i(x^i) > u_i(y)$  for all  $y \in X \setminus \{x^i\}$ , (ii)  $y < z < x^i$  implies  $u_i(z) > u_i(y)$ , and (iii)  $x^i < z < y$  implies  $u_i(z) > u_i(y)$ . If the game goes on indefinitely and no policy is enacted, the voter's utility,  $u_i(\emptyset)$ , equals  $\phi$ , where  $\phi < 0$ .

At any given point in time, viable candidates for office can be identified, i.e., successful candidates rarely appear out of thin air. Political success is usually obtained through political parties that are hierarchical organizations and it may take years of service to rise to the top of the party. Exceptions exist but they generally rely on other resources, such as money or name recognition, both of which are not at the average citizen's disposal. I assume, therefore, that at the beginning of the game there is a set of  $k$  candidates

<sup>9</sup>The same claim can be made about legislative elections in France. See, e.g., Blais and In-dridason (2007) and Golder (2000).

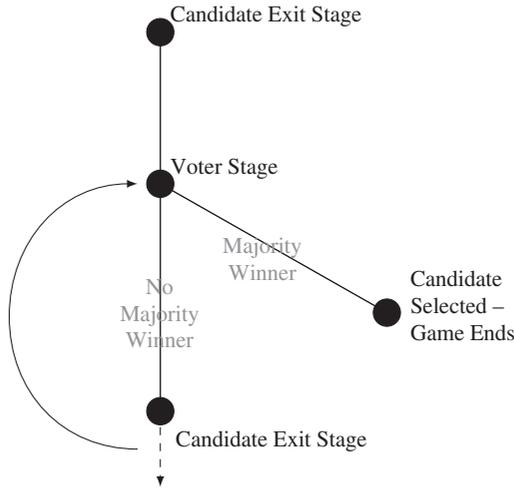


Figure 1. The sequence of the game.

$K \subset N$  that can run for office. The candidates are motivated by the prospects of holding office and/or influencing policy outcomes. Candidate  $k$ 's most preferred policy is denoted  $x^k \in X$  and it is assumed that candidates are unable to make credible commitments to implement any other policy. Assume, furthermore, that no two candidates share the same platform. Throughout I assume that campaigning is costly.

Each period of the game consists of two stages. The sequence of play is sketched in Figure 1. At the first stage the candidates decide between staying in the race or exiting. Let candidate  $k$ 's action at time  $t$  be  $e_k^t \in \{0, 1\}$  where 0 denotes exit and 1 denotes a decision to stay in the race. Denote the set of candidates in the running at time  $t$  as  $K^t$ , i.e., if candidate  $k$  exits at time  $t$  then  $k \notin K^t$ . I assume that the candidates incur a cost,  $c_k$ , each period they stay in the race. The cost can be interpreted as the cost of campaigning, including opportunity cost.

At the second stage the voters cast a vote for one of the candidates. Voter  $i$ 's action at time  $t$  is denoted  $b_i^t \in K^t$ . Let  $B^t = \prod_{i \in N} K^t$  be the set of outcomes in the voter stage at time  $t$ . Let  $v_{k,K^t}$  denote  $k$ 's vote share in a race between  $k$  and the members of  $K^t \setminus k$ , where  $K^t$  is the set of candidates that have not exited at time  $t$ . For example, if  $K^t = \{j, k\}$  then  $v_{k,K^t} = v_{k,j}$ . In section 4, I consider strategic voting and show that the set of possible equilibria under iterated elimination of weakly dominated strategies is a subset of the equilibria under sincere voting.

The votes are tallied following each poll. Candidate  $j$ 's vote share equals

$$v_j^t = \frac{|\{i \in N | b_i^t = j\}|}{\sum_{l=1}^k |\{i \in N | b_i^t = l\}|}$$

If  $v_{j,K^t}^t > 1/2$  for some  $j \in K$ , candidate  $j$  is declared the winner of the election and the game ends. If  $v_{j,K^t}^t \leq 1/2$  for all  $j \in K$  then no candidate can be declared a winner and the game continues for another period. This goes on until a winner emerges, whether as a result of voter coordination or candidate exit.

In each period candidate  $k$  decides whether to stay in the race or to exit. The candidate's action is  $e_k \in \mathbb{N}$ , where  $e_k$  denotes the time of exit, i.e., there is no re-entry. Let  $e_k^t = 0$  if  $t \geq e_k$  and  $e_k^t = 1$  else. Then  $E^t = \{e_1^t, e_2^t, \dots, e_k^t\}$  is the set of outcomes in the candidate stage at time  $t$  and  $A^t = B^t \times E^t$  the set of outcomes in the candidate and voter stages. Let the set of outcomes of the game be  $A = \prod_{t=1}^{\infty} A^t$  and  $a \in A$  be an action profile. The candidates' utility functions,  $w_k$ , are of the following form:

$$w_k(x, a) = \gamma [2v_{k,K^t}(a)] + (1 - \gamma)u_k(x(a)) - \sum_{t=1}^{\infty} e_k^t c_k. \tag{1}$$

In the first term,  $\gamma \in [0, 1]$  is the utility that the candidate receives from getting into office, and  $[2v_{k,K^t}(a)]$  is an indicator of whether candidate  $k$  won or not (recall  $v_{k,K^t}$  is candidate  $k$ 's share of the vote in round  $t$  and  $[x]$  denotes the largest integer smaller than  $x$ ).<sup>10</sup> The profile of utility functions is single-peaked. The function  $u_k$  is strictly concave in the enacted policy,  $x(a)$ . The function has a maximum at the candidate's most preferred policy,  $x^k$ , and  $\gamma + u_k(x^k) - u_k(x^j) > c_k, \forall j \in (K \setminus \{k\}) \cup \{SQ\}$ , where  $x^{SQ}$  is the reversion policy or the status quo.<sup>11</sup> The inequality implies that every candidate prefers running and winning to exiting and losing regardless of who the winning candidate is. I normalize  $u_k(x^k) = 0$  but occasionally I include  $u_k(x^k)$  in the expressions if it provides better intuition. The relative importance of policy to office (and the cost of running) is parameterized by  $\gamma$ . The final term reflects the cost of running where  $c_k$  denotes the cost of staying on the ballot in each round and  $e_k^t$  captures whether the candidate stayed in the running in each round.

I begin by stating, without proof, two lemmas that are standard results in the literature. The first lemma demonstrates that if the profile of the voters' preferences,  $\{u_i(x)\}_{i \in N}$ , is single-peaked, there exists a candidate that obtains at least half the vote share in a pairwise contest against any other candidate, i.e., a *weak Condorcet winner* exists. Define  $K^C = \{k \in K | v_{k,j} \geq 1/2, \forall j \in K\}$  as the set of weak Condorcet winners. It is shown that  $K^C$  contains at most two candidates. The second lemma shows that any candidate belonging to the set of weak Condorcet winners,  $K^C$ , is strictly majority preferred to any candidate not belonging to the set of weak

<sup>10</sup>Note that "the greatest integer smaller than  $x$ " implies that  $[x] = 0$ . This breaks away from conventional usage but is adopted here because a strict majority of votes is needed to win office.

<sup>11</sup>Relaxing this assumption allows equilibria in which a non-Condorcet winner wins with certainty. An interesting implication, that is not explored, is that the median-voter may be worse off when the candidates have policy preferences.

Condorcet winners. Although the voters' utility functions are single-peaked over the domain of the policy space their preferences are not necessarily single-peaked over the set of candidates, i.e., a voter may be indifferent between his two most preferred candidates.

**Lemma 1.** If  $\{u_i(x)\}_{i \in N}$  is single-peaked on  $X$ , and  $K$  is a set of candidates offering distinct policy platforms, there exists a candidate,  $k \in K$ , such that  $v_{k,j} \geq 1/2$ ,  $\forall j \in K$ . Moreover,  $|K^C| < 3$ .

**Lemma 2.** If  $\{u_i(x)\}_{i \in N}$  is single-peaked on  $X$ , and  $K$  is a set of distinct candidates,  $K^C = \{k \in K | v_{k,j} \geq 1/2, \forall j \in K\}$ , then  $v_{k,j} > 1/2$ ,  $\forall j \in K$  for all  $k \in K^C$  and all  $j \in K \setminus K^C$ .

### 2.1 Policy and Office Motivated Candidates

The motivations of politicians can be expected to influence their behavior.<sup>12</sup> On one hand, one might expect candidates to act in fundamentally the same ways whether they are motivated by office or policy – to influence policy one must get elected and to get elected one must at least pretend to care about policy. On the other hand, policy and office provide slightly different incentives. When candidates are motivated by a desire to influence policy their chances of doing so may be better by handing the victory over to another candidate by exiting, than persisting and possibly losing to a candidate with less similar policy preferences. The candidates' motivations thus influence their incentives to coordinate electorally. The relative importance of policy and office is considered in greater detail below.

At most two candidates do not exit in the first round in the subgame-perfect equilibria of the game. However, the existence of an equilibrium where two candidates stay in the race relies on knife-edge conditions. There always exists an equilibrium where a candidate in the Condorcet set emerges as the winner. Propositions 1 and 2 show the existence of such equilibria when the Condorcet set is either a singleton or has two elements, respectively.

**Proposition 1.** Suppose the Condorcet set consists of a single candidate,  $K^C = \{k^*\}$ . Then there exists an equilibrium in which the Condorcet winner stays in and all other candidates exit right away.

*Proof.* Consider the above strategy profile. Candidate  $k^*$ 's action is clearly optimal. If candidate  $k \in K \setminus \{k^*\}$  deviates and stays in the race, then there will be a two-candidate contest. Then  $v_{k^*,k} > 1/2$ , as  $k^* \in K^C$ , candidate  $k$  will lose. Thus, immediate exit,  $e_k = 1$ , is optimal for candidate  $k$ . ■

<sup>12</sup>For example, Calvert (1985), Mitchell (1987), and Wittman (1983) focus on policy divergence. Callander and Wilke (2002) consider competition between candidates who value policy and office differently.

**Proposition 2.** Suppose the Condorcet set contains two candidates,  $K^C = \{k^-, k^+\}$ . There always exists an equilibrium where one member of the Condorcet,  $k^* \in K^C$ , stays in the race and all others exit. If the candidates' costs are sufficiently low there also exist equilibria where both candidates stay in the race, all others exit, and the winner is determined by a lottery.

*Proof.* First, suppose that candidate  $k^* \in K^C$  stays in while all other candidates exit immediately. As  $k^* \in K^C$ , any  $k^* \notin K^C$  would lose if they entered. Assume, without loss of generality, that  $k^- = k^*$ . As  $|K^C| = 2$  it must be the case that the median voter,  $m \in N$ , is indifferent between  $k^-$  and  $k^+$ , i.e.,  $u_m(k^-) = u_m(k^+)$ . If voter  $m$ 's strategy is to vote for  $k^-$  when  $K^1 = K^C$  then  $k^+$  will not benefit from staying in the race. Hence, no beneficial deviations exist for any of the candidates. Now, suppose both the candidates in  $K^C$  stay in but all the other candidates exit. The candidates both prefer to stay in and face a lottery where  $k^-$  wins with probability  $p$  and  $k^+$  wins with probability  $1 - p$  if the following conditions holds: (i)  $p(1 - \gamma)u_{k^-}(x^{k^-}) < p[\gamma + (1 - \gamma)u_{k^-}(x^{k^-})] - c_{k^-}$  and (ii)  $(1 - p)(1 - \gamma)u_{k^+}(x^{k^+}) < (1 - p)[\gamma + (1 - \gamma)u_{k^+}(x^{k^+})] - c_{k^+}$ . If the two conditions cannot be satisfied for some  $p \in (0, 1)$  then one of the candidates will prefer exit to staying in. Assume  $p$  satisfies the conditions. Assume  $m$  casts his vote for  $k^-$  with probability  $p$  and  $k^+$  with probability  $1 - p$  when  $K^1 = K^C$ . By definition,  $p$  guarantees that both  $k^-$  and  $k^+$  prefer the lottery to exiting immediately. If  $k \notin K^C$  decides not to exit then assume  $m$  casts his vote for  $k^-$  if  $x^k > x^{k^+}$  and  $k^+$  else. Then either  $k^-$  or  $k^+$  wins outright if  $k$  enters, reducing his utility by at least  $c_k$ . It is easy to verify that if more than one voter is indifferent between  $k^-$  and  $k^+$  similar equilibria can be constructed. ■

By Proposition 2, two types of equilibria are possible if the Condorcet set contains two candidates. In the first type, one of the candidates in the Condorcet set stays in while all other candidates exit at  $t = 1$ . In the second type, both the candidates stay in the race and all other candidates exit at  $t = 1$ . The existence of the latter equilibrium depends on both candidates having a sufficiently high probability of winning to offset their costs of running.

Another two candidate races, involving candidates that do not belong to the Condorcet set, are possible but, as before, they require ties between candidates. Define  $K^m(j, l)$  as the set of candidates whose ideal policies lie between the ideal policies of candidates  $j$  and  $l$ :

$$K^m(j, l) = \{k \in K | x^j < x^k < x^l\}. \tag{2}$$

Define  $t^k(j, l, p)$  as the maximum number of rounds a candidate located in between candidates  $j$  and  $l$  is willing to run when the alternative is to exit and

face an outcome that is a lottery,  $p \in (0, 1)$ , over the ideal policies of candidates  $j$  and  $l$ :

$$t^k(j, l, p) = \max\{t \in Z \mid \gamma + (1 - \gamma)u_k(x^k) - (1 - \gamma)(pu_k(x^j) + (1 - p)u_k(x^l)) \geq tc_k\}. \quad (3)$$

**Proposition 3.** Assume the following conditions are satisfied: (a) candidates  $j$  and  $l$  tie by the majority preference relation, (b) any candidate  $k$  with a platform between candidates  $j$  and  $l$  will carry less than a majority of the vote in a three-candidate contest, and (c) candidate  $k$ 's cost of running is high relative to that of candidates  $j$  and  $l$ . Formally, if

- (a) there exists a pair  $\{j, l\} \in K \setminus K^C \times K \setminus K^C$  such that  $v_{j,l} = 1/2$ ,
- (b)  $v_{k,\{j,l\}} \leq 1/2, \forall k \in K^m$ ,
- (c) there exists  $p \in (0, 1)$  such that:
  - (i)  $(1 - \gamma)u_j(x^k) < p[\gamma + (1 - \gamma)u_j(x^j)] + (1 - p)(1 - \gamma)u_j(x^l) - t^k(j, l, p)c_j$  and
  - (ii)  $(1 - \gamma)u_l(x^j) < (1 - p)[\gamma + (1 - \gamma)u_l(x^l)] + p(1 - \gamma)u_l(x^j) - t^k(j, l, p)c_l$ ,

then there exists a two-candidate equilibrium where candidates  $j$  and  $l$  stay in and all other candidates exit immediately.

*Proof.* The actions of candidates  $j$  and  $l$  are optimal given condition (c), i.e., they prefer to pay the cost of running and facing a lottery over the two possible outcomes,  $x^j$  and  $x^l$ , to exiting. Let  $m \in \{i \in N \mid u_i(x^j) = u_i(x^l)\}$ , which is non-empty by  $n$  odd. Without loss of generality, assume  $j < l$ . If  $K^1 = \{j, l, k\}$  then let  $b_m = j$  if  $x^k < x^j$  and  $b_m = l$  if  $x^k > x^j$ . Then staying out of the race is optimal for  $k \notin K^m \cup \{j, l\}$ . For candidate  $k \in K^m$  exiting at  $t = 1$  is a best response because  $v_{k,\{j,l\}} \leq 1/2$  and by definition of  $t(j, l, p)$ , i.e., candidate  $k$  may be able to win but only at an excessive cost. ■

The equilibrium described in Proposition 3 is not necessarily unique – multiple pairs of candidates may satisfy the conditions of the proposition. By Proposition 3, two-candidate equilibria that do not involve the candidates most preferred by the median voter may exist. Hence, policy divergence is possible even in the presence of centrist alternatives.<sup>13</sup>

Three factors are important for the existence of the equilibrium. First, there must exist a pair of candidates that are not majority preferred to one another, i.e., the median voter must be indifferent between the candidates.

<sup>13</sup>Note that I have assumed throughout that candidates cannot share a policy platform. Hence, policy convergence is not possible in the model by fiat. However, the proof of the result does not depend on the assumption.

Second, the policy platforms of the two candidates cannot be too extreme – if that were the case, a third candidate, located between them, could garner a majority of the vote by staying in the race. Exactly how extreme “too extreme” is, depends on the distribution of the voters’ preferences. The candidates’ platforms must be close to the median when the density of voters’ ideal policies around  $x^m$  is great. Osborne and Slivinski (1996) obtain a similar result in the context plurality rule elections, i.e., the two candidates cannot be located so far apart as to allow a centrist candidate to win a plurality of the vote. Here, Proposition 3 allows for greater divergence of the candidates’ positions because the centrist candidate must win a majority of the vote to bring the contest to an end. In Osborne and Slivinski (1996) the conditions on divergence are not sufficient because a centrist candidate facing low costs of campaigning may benefit from staying in the race.<sup>14</sup>

Finally, existence of the equilibrium depends on the willingness of the candidates in  $K^m$  to run, which in turn depends on their cost of running and the intensity of their policy preferences [i.e., the first derivative of  $u_k(x)$ ]. The two candidates must be willing to stay in the race long enough to deter more centrist candidates from staying in the race. This condition is satisfied if the centrally located candidates’ cost of running is high and the candidates place a high value on obtaining a favorable policy outcome, which is a function of the value of office,  $\gamma$ , and the first derivative of the candidates’ utility function. Thus, relative indifference over policy generally disadvantages non-centrist candidates.<sup>15</sup>

The above propositions have established the possibility of one- and two-candidate equilibria. The following proposition shows that the only equilibria of the game are the ones shown in Propositions 1–3.<sup>16</sup> The intuition behind Proposition 4 is particularly simple. If a candidate does not emerge as the winner, she will prefer to exit as soon as possible. In equilibrium, then, every candidate must have a positive probability of winning, which rules out pure strategy equilibria with more than two candidates.

**Proposition 4.** The equilibria described in Propositions 1–3 are the only pure strategy equilibria of the game.

*Proof.* First note that the game must end in finite time because running for office is costly, i.e., all the candidates prefer exiting immediately to running

<sup>14</sup> Osborne and Slivinski (1996) also show that the two candidates cannot have identical positions in equilibrium. It is assumed here that candidates do not share positions but it can be shown that the two-candidate equilibrium does not depend on diverging positioning of candidates.

<sup>15</sup> Interestingly this also suggests that uncompromising and programmatic extreme parties will be more likely to survive than accommodating extreme parties.

<sup>16</sup> The assumption of an odd number of voters is not entirely innocuous here. Because Lemma 2 fails when  $n$  is even it is possible to construct an equilibrium in which one of the candidates in the Condorcet set and another candidate stay in while all other candidates exit. The equilibrium is very similar to the equilibrium in Proposition 2 and therefore adds little to the results.

for office for an infinite number of periods. Suppose that some  $j \in K$  wins a majority of the vote at time  $t$ . Then, in equilibrium, we must have  $e_k^1 = 0$ ,  $\forall k \in K \setminus \{j\}$  because campaigning is costly. If  $j \notin K^C$  then  $k \in K^C$  can win by staying in the race,  $e_k^1 = 1$ , as  $v_{k,j} > 1/2$  if  $k \in K^C$  and  $j \notin K^C$ . Hence, no equilibria can exist where  $j \notin K^C$  wins with certainty. Furthermore, this rules out the possibility of any equilibria in which three or more candidates do not exit in the first period, i.e., no voter can be indifferent between more than two candidates and consequently at most two candidates have a positive probability of winning. It remains to rule out alternative two-candidate equilibria. It is easy to see that if the two candidates do not tie (in terms of the number of voters who have strict preference over the candidates), as in Propositions 2 and 3, one of them wins an outright victory. Then optimality demands that the other candidate exit at  $t = 1$ . ■

Generally it does not make a substantial difference whether the candidates seek office or policy influence. The types of equilibria are exactly the same whether the candidates are motivated by policy or office – candidate preferences merely influence the conditions under which two-candidate equilibria may occur. One reason for considering policy-motivated candidates is the possibility of a strategic candidacy – the possibility that a candidate enter the fray to increase some other candidate’s chance of winning. In contrast with the literature that has considered strategic candidacy (Besley and Coate, 1997; Dutta et al., 2001; Osborne and Slivinski, 1996) no equilibria where candidates run to facilitate the election of another candidate exist here. In Osborne–Slivinski, e.g., a three-candidate equilibrium, in which one of the candidates has no possibility of winning, may exist because the “spoiler” candidate draws disproportional support from the candidate that she prefers less. In Osborne–Slivinski this possibility exists because the elections are conducted under plurality rule. Under majority rule, a “third” candidate cannot level the playing field in this manner because her presence will reduce one of the candidates’ chances of winning to zero, in which case it is optimal for the candidate to exit in the first period. Therefore, “spoiler candidates” cannot exist in equilibrium. Rather, as Proposition 3 shows, potential “spoiler candidates” can only rule out equilibria in which median divergence occurs, i.e., when the outcome of the election is a lottery between candidates that are not in the Condorcet set.

The results obtained above bear resemblance to some well-known results in the literature. The equilibrium number of candidates never exceeds two, the number of candidates predicted by Duverger’s Law. The conditions for two-candidate equilibria require the median voter to be indifferent between the two candidates. Thus, equilibria in which only one candidate runs for office appear more “reasonable”. Formal derivations of Duverger’s Law have assumed that strategic voter behavior drives electoral coordination. A similar outcome occurs, however, in the model presented here even though

the voters are assumed to act sincerely. We obtain results analogous to Duverger's Law when only the candidates can influence the process of electoral coordination.<sup>17</sup> The model suggests a causal argument that is quite different from the one that usually accompanies Duverger's Law. That is, Duverger's Law holds in the absence of strategic voting if (potential) candidates for office act strategically.

The model also shows *which* candidates stay in the race, i.e., which policy platforms the voters can choose between. The results extend the median voter theorem to a dynamic setting. The median voter theorem provided here is, however, a "weak" version of the median voter theorem for two reasons. First, because the candidates cannot make credible commitments, there is no guarantee that the winning candidate's policy platform will coincide with the median voter's ideal policy. However, except for the equilibria described in Proposition 3, the median voter's most preferred candidate wins the election, i.e., the median voter obtains his best possible outcome given the set of viable candidates. Second, in some instances "divergent" two-candidate equilibria may exist, as shown in Proposition 3. In these equilibria the median voter's most preferred candidate exits the contest in the first period. While such "divergent" equilibria are possible, they are subject to knife-edge conditions.

Propositions 1–4 achieve two of the three goals set out at the beginning. The model simultaneously predicts the number of candidates/parties and their location in the policy space. The third goal was to offer an insight into why coordination failure occurs, i.e., instances in which the number of candidates exceeds the expectations established by Duverger's Law. Changing the informational assumptions of the model could clearly give rise to equilibria in which fewer candidates exit. While uncertainty about the outcome undoubtedly has some role in explaining coordination failure, empirically it appears that the actors involved are often well aware of the consequences of their actions. In the context of French legislative elections it appears, for example, problematic to argue that the parties of the right have not realized the dangers of offering several candidates in each constituency – especially as the parties have repeatedly paid the price for failing to coordinate their actions.<sup>18</sup> Hence, it appears that uncertainty is not the only factor at work.

One reason coordination may fail is that while the candidates may prefer coordination to not coordinating, they have preference about the form that

<sup>17</sup>It is possible to claim that the results are analogous to Duverger's Law. However, most other models allow neither for exit nor abstention, and two-candidate equilibria in these models thus exist largely by fiat. The claim here should be understood to mean that the important aspect of Duverger's Law is not the exact number of parties but the strong downward pressure on the number of parties.

<sup>18</sup>Admittedly, the merger of the main center-right parties into the UMP in 2002 suggests that the parties eventually realized the necessity of coordinating their actions.

the coordination takes, i.e., which candidate exits. Hence, the candidates find themselves engaged in a war of attrition, each hoping that the other will exit. In this scenario the candidates may adopt strategies that anticipate the possibility that the other candidates will exit with some probability. In the next section I consider the mixed strategy equilibria of the game.

### 3. MIXED STRATEGY EQUILIBRIA

Mixed strategy equilibria offer an explanation of why coordination failure may occur. If the candidates play mixed strategies there is a positive probability that no candidate exits and that the contest goes on for another round with additional costs for the candidates. While the mixed strategy equilibria of the game are unstable, they offer the advantage that they respect the symmetric circumstances of the candidates. That is, suppose either of two candidates will win the election if they coordinate their actions (one candidate exits). Focusing only on the pure strategy equilibria of the game amounts to assuming away the problem of coordination. Thus, pure strategy equilibria capture the strong incentives to coordinate but they leave out an important element, namely, whether coordination can be achieved efficiently.<sup>19</sup>

I restrict my attention to Markov-perfect equilibria, in which the candidates' strategies depend only on the set of candidates that are still in the running. Hence, the dependence on the history of play can be summarized as a state variable that corresponds to the set of candidates at time  $t$ , i.e.,  $K^t$ . The number of possible states of the game depends on the number of candidates. For any game there is a set,  $H$ , of possible states  $\eta \in H$ . When  $|K| = 4$ , and the candidates are linearly ranked, there are five states (corresponding to  $\{1, 2, 3, 4\}$ ,  $\{1, 2, 3\}$ ,  $\{1, 2, 4\}$ ,  $\{1, 3, 4\}$ , and  $\{2, 3, 4\}$ ). A "two-candidate state" never occurs when the candidates are linearly ranked as it would imply that the other candidates must have exited in the previous period and the subsequent ballot must, therefore, have ended the game.

For the sake of tractability I only consider office-seeking candidates, i.e., the candidate's payoff (net of cost) equals zero if he loses and one if he wins. I furthermore restrict my attention to the case where the number of viable candidates is three. Each candidate's probability of winning depends on the probability of exit for each subset of the candidates. Solving for the equilibrium becomes increasingly cumbersome as the number of candidates increases as the number of possible outcomes increases exponentially with the number of candidates.

To illustrate the problem, suppose there are three candidates that are linearly ordered by the majority preference relation. Let Candidate  $k$  be the

<sup>19</sup>See Farrell (1987) for a similar argument in the context of the entry of firms into an industry.

$k$ th-ranked candidate. Assume  $v_{1.23} \leq 1/2$ . Candidate 1 wins if some other candidate exits but Candidate 2 wins if Candidate 1 exits. Finally, Candidate 3 only wins if Candidate 1 *and* Candidate 2 exit simultaneously. Intuitively then, the probability that Candidates 1 and 2 exit must be relatively high for Candidate 3 to be indifferent between staying in and exiting. Similarly, the probability of exit for Candidate 1 has to be fairly high if Candidate 2 is indifferent between running and exiting.<sup>20</sup> Writing down the equilibrium conditions becomes increasingly complex if there are more candidates because the number of contingencies to consider increases rapidly as the number of possible states goes up.

Here I consider the case where the candidates are linearly ranked, i.e., the median voter has strict preference over any pair of candidates. To allow for mixed strategies redefine candidate  $k$ 's action spaces as the probability of exit at time  $t$ ,  $p_k^t \in [0, 1]$ . As I restrict my attention to Markov-perfect equilibria, the dependence of the strategy on time can be dropped, i.e.,  $p_k = p_k^t$ ,  $\forall t \in T$ . Let  $V_k^t$  denote candidate  $k$ 's continuation value, i.e., the expected value of staying in the game, at time  $t$ .

**Proposition 5.** If the candidates are linearly ranked, i.e.,  $v_{1.2} > 1/2$ ,  $v_{2.3} > 1/2$ ,  $v_{1.3} > 1/2$ , and  $c_3 \leq c_1 c_2$  there exists a stationary equilibrium such that all the candidates stay in the race with a positive probability. The candidates' probabilities of exit equal  $p_1 = c_2$ ,  $p_2 = c_3/c_2$ , and  $p_3 = (c_1 c_2 - c_3)/(c_2 - c_3)$ .

*Proof.* Candidate 1 must be indifferent between exiting and staying in at time  $t$  if he plays a mixed strategy. The following equalities must therefore hold:

$$\begin{aligned} V_1^{t+1}(1 - p_2)(1 - p_3) + (1 - tc_1)[1 - (1 - p_2)(1 - p_3)] \\ = -(t - 1)c_1, \end{aligned} \tag{4}$$

and

$$\begin{aligned} V_1^{t+2}(1 - p_2)(1 - p_3) + (1 - (t + 1)c_1)[1 - (1 - p_2)(1 - p_3)] \\ = -tc_1. \end{aligned} \tag{5}$$

The continuation value for Candidate 1 at time  $t + 1$  depends on the strategy he employs and can be written as:

$$\begin{aligned} V_1^{t+1} = p_1(-2c_1) + (1 - p_1)[V_1^{t+2}(1 - p_2)(1 - p_3) \\ + (1 - (t + 1)c_1)(1 - (1 - p_2)(1 - p_3))]. \end{aligned} \tag{6}$$

Solving for the continuation values in the first two expressions and substituting into the last one yields  $1 - [(1 - c_1)/((1 - p_2)(1 - p_3))] = 0$ .

<sup>20</sup>It is also possible that two candidates tie at the top or at the bottom if the median voter is indifferent between the candidates. The results for these cases are available on request from the author.

Turning to Candidate 2, he must be similarly indifferent between exiting and staying in. Thus,

$$\begin{aligned} V_2^{t+1}(1-p_1)(1-p_3) + p_1(1-tc_2) + (1-p_1)p_3(-tc_2) \\ = -(t-1)c_2 \end{aligned} \quad (7)$$

and

$$\begin{aligned} V_2^{t+2}(1-p_1)(1-p_3) + p_1(1-(t+1)c_2) \\ + (1-p_1)p_3(-(t+1)c_2) = -tc_2. \end{aligned} \quad (8)$$

The continuation value for Candidate 2 at time  $t+1$  equals:

$$\begin{aligned} V_2^{t+1} = p_2(-c_2) + (1-p_2)[V_2^{t+2}(1-p_1)(1-p_3) + p_1(1-(t+1)c_2) \\ + (1-p_1)p_3(-(t+1)c_2)]. \end{aligned} \quad (9)$$

Solving as before for the continuation values and making the substitutions yields  $p_1 = c_2$ . Finally, if Candidate 3 is indifferent between running and exiting, the following equalities hold:

$$\begin{aligned} V_3^{t+1}(1-p_1)(1-p_2) + p_1p_2(1-tc_3) + [(1-p_1)p_2 + (1-p_2)p_1](-tc_3) \\ = -(t-1)c_3, \end{aligned} \quad (10)$$

and

$$\begin{aligned} V_3^{t+2}(1-p_1)(1-p_2) + p_1p_2(1-(t+1)c_3) \\ + [(1-p_1)p_2 + (1-p_2)p_1](-(t+1)c_3) = -tc_3. \end{aligned} \quad (11)$$

The continuation value for Candidate 3 equals:

$$\begin{aligned} V_3^{t+1} = p_3(-(t-1)c_2) + (1-p_3)[V_3^{t+2}(1-p_1)(1-p_2) \\ + p_1p_2(1-(t+1)c_3) + [(1-p_1)p_2 \\ + (1-p_2)p_1](-(t+1)c_3)]. \end{aligned} \quad (12)$$

Substituting as before yields  $p_2 = c_3/p_1$ , or  $p_2 = c_3/c_2$  as  $p_1 = c_2$ . It only remains to solve for  $p_3$  but above it has been shown that  $1 - [(1-c_1)/(1-p_2)(1-p_3)] = 0$ . Substituting in for  $p_2$ , and solving for  $p_3$ , yields  $p_3 = (c_1c_2 - c_3)/(c_2 - c_3)$ . Finally, it is easy to check that if, and only if,  $c_3 \leq c_1c_2$  are both  $p_2$  and  $p_3$  well-defined probabilities. ■

Proposition 5 shows that a stationary equilibrium in mixed strategies exists if Candidate 3's cost of running is lower than the multiple of the other candidates' costs of running. Figures 2 and 3 show the relative size of the

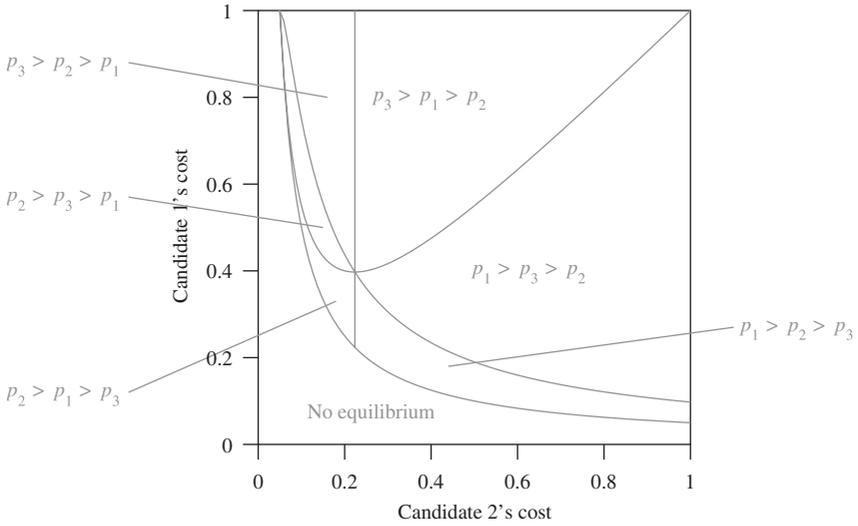


Figure 2. Probability of exit: Candidate 3's cost of running = 0.05.

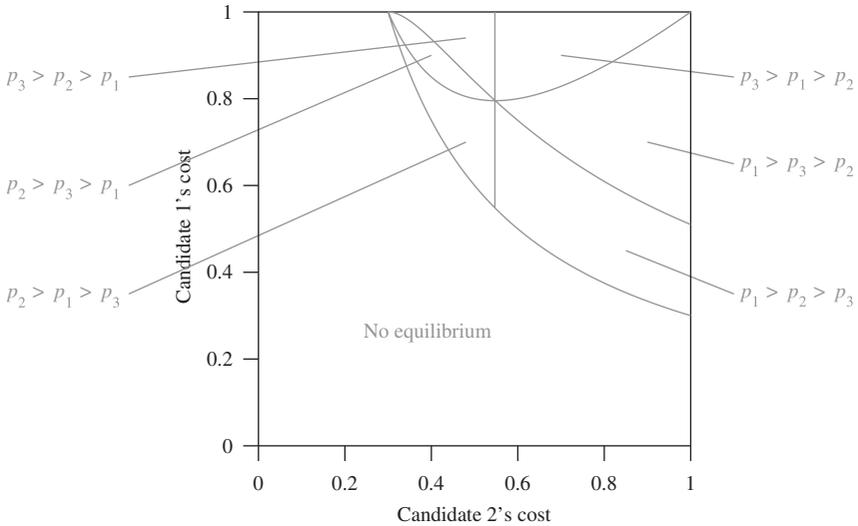


Figure 3. Probability of exit: Candidate 3's cost of running = 0.3.

candidates' probabilities of exit for different values of  $c_1$  and  $c_2$  while holding  $c_3$  fixed (0.05 and 0.3). Candidate 2 has the lowest probability of exit for most cost combinations. Candidate 1 is most likely to exit for a large portion of the cost combinations, as is Candidate 3.

It can be seen from the figures that Candidate 1 goes from having the highest probability of exit to having the second highest as her cost increases. Candidate 1's probability of exit depends only on Candidate 2's cost but it also influences the equilibrium strategies of the other two candidates. Candidate 2's probability of exit also drops as her cost goes up, but in her case the cost has a direct impact on her own strategy, as can be seen in Proposition 5. Similar statements can be made about Candidate 3, although this is not quite as easy to spot from the figures. The comparative statistics are easily obtained:  $\partial p_1/\partial c_2 > 0$ ,  $\partial p_2/\partial c_2 > 0$ ,  $\partial p_2/\partial c_3 > 0$ ,  $\partial p_3/\partial c_1 > 0$ ,  $\partial p_3/\partial c_2 > 0$ , and  $\partial p_3/\partial c_3 > 0$ .

The electoral support of the parties plays only a limited role in the mixed strategy equilibrium, i.e., the exit probabilities are only influenced by the parties' ranks by the majority preference relation. The candidates' cost of running, on the other hand, influences both the possibility of coordination failure and the candidates' exit probabilities.

#### 4. STRATEGIC VOTING

So far it has been assumed that voters do not act strategically but simply vote for the candidate they like the most. Strategic voting gives rise to additional equilibria – some of which are multi-candidate equilibria where the winner is not the same as in the equilibrium under sincere voting. In this section I consider an example of an equilibrium where the outcomes under sincere and strategic voting differ. Subsequently I argue that the existence of such equilibria places great demands on voters' ability to coordinate their actors. I then show that a further equilibrium refinement, iterated elimination of dominated strategies, reduces the set of equilibria under strategic voting to the set of equilibria under sincere voting.

First, note that the sincere equilibrium strategies in section 1 remain in equilibrium under strategic voting, i.e., the voters have no incentive to defect from these strategies. The question then is why the sincere voting equilibria deserve greater attention than the additional equilibria that exist under strategic voting.

It is instructive to consider what the additional equilibria under strategic voting look like. Assume  $K = \{1, 2, 3, 4, 5, 6\}$ ,  $K^C = \{3\}$ , and  $x^i < x^l$  if and only if  $j < l$ . If the candidates care sufficiently about policy and the costs of running are low then the following path of play can be supported in equilibrium: no candidate exits in the first round and all the voters cast their vote for candidate 4. As the solution concept is subgame-perfect Nash equilibrium it is sufficient to consider deviations by individual candidates (and voters). The equilibrium path of play is supported by the following voter strategies:

$$b_i^1 = \begin{cases} 2, & \text{if } K \setminus K^1 \subset \{4, 5, 6\} \\ 4, & \text{if } K^1 = K, \\ 5, & \text{if } K \setminus K^1 \subset \{1, 2, 3\} \end{cases} \quad \forall i \in N. \quad (13)$$

For example, if candidate 4 exits from the race the voters will elect candidate 2. The voters' strategies punish a candidate that exits by electing a candidate whose policy platform lies further away from the exiting candidate's preferred policy. Staying in the race is a best response for each of the candidates if the cost of running is low enough. The candidate's punishment for exiting equals the difference in the utility from having Candidate 4 elected and the utility from having the "punishment" candidate elected. The candidate will prefer to stay in the race if the difference is greater than her cost of running. The voters' strategies are optimal because the voters are never pivotal and their voting strategies are not weakly dominated. The construction of the equilibrium takes advantage of the limited bite that elimination of weakly dominated strategies has in multi-candidate contests, i.e., it only rules out the possibility that the voters cast votes for their least preferred candidate.

The equilibrium sketched out above has some unattractive qualities. First, as I argue at the outset of the paper, voters are generally at a disadvantage when it comes to coordinating their actions. Empirical studies of strategic voting have estimated that generally only about 5% of the voting population votes strategically (Alvarez and Nagler, 2000; Blais, 2002). The example above clearly requires a much higher level of sophistication on behalf of voters than simply deserting "third" candidates, which is what the empirical studies have sought to measure.<sup>21</sup> Second, the voters coordinate on an equilibrium in which a majority of voters are worse off than if they voted sincerely. Of course, welfare comparisons of this sort are not valid criteria for equilibrium selection. However, it is not clear why voters would adopt much more complicated, and presumably more costly, strategies to obtain an inferior outcome.<sup>22</sup>

The multiplicity of voting equilibria under strategic voting is not unique to the model considered here. On the contrary, multiple equilibria exist in most multi-candidate models. The problem boils down to the fact that the elimination of dominated strategies, the standard equilibrium refinement in voting games, is too weak in the sense that more would be expected of rational voters. De Sinopoli and Turrini (1999) and Dhillon and Lockwood (2001) consider a stricter condition, iterated elimination of dominated strategies, in the context of Besley and Coate's (1997) citizen-candidate model. The refinement only proves to be useful in considering equilibria with four or more candidates. However, the iterated elimination of dominated

<sup>21</sup>In some sense there is disjuncture in the literature on voting – our theories of voting have a great difficulty explaining why people vote when voting is costly, yet we are often willing to assume that the voters are willing to go to great (and costly) lengths to vote strategically.

<sup>22</sup>It is possibly to imagine a scenario in which voters would behave in this manner. For example, voters might view coalescing parties as behaving opportunistically, depriving the voters of choice, and be willing to punish such behavior. This, of course, raises a number of other questions regarding voter rationality.

strategies has considerably more bite under majority rule and single-peaked preferences than under plurality rule.

The elimination of weakly dominated strategies only prevents voters from casting their votes for the candidate that they like the least. The iterated elimination of dominated strategies goes a step further by assuming that each voter is aware that other voters behave in the same manner. Hence, the process of elimination is repeated iteratively. Formally, define a sequence of sets of voting profiles  $\{B^0, B^1, B^2, \dots\}$  in the following manner.<sup>23</sup> As in section 2, let  $B^0 = \prod_{i \in N} K$  be the set of unrestricted voting profiles. Let  $B^z = \prod_{i \in N} B_i^z$ , where  $B_i^z$  is the set of actions undominated under  $B^{z-1}$ . An action,  $b_i$  is undominated under  $B$  if there does not exist  $b'_i$  such that  $u_i(b'_i, b_{-i}) \geq u_i(b_i, b_{-i})$ ,  $\forall b_{-i} \in B_{-i}$ , and  $u_i(b'_i, b_{-i}) > u_i(b_i, b_{-i})$  for some  $b_{-i} \in B_{-i}$ . Hence,  $B^1$  corresponds to the set of voting profiles after the elimination of weakly dominated strategies while the limit of the sequence,  $B^\infty$ , corresponds to the iterated elimination of dominated strategies.

The assumption of iterated elimination of dominated strategies is a strong assumption. The voters must be informed about the distribution of voter preferences and have the capability to figure out when voting for a particular candidate becomes a dominated strategy. It is thus clear that the assumption places considerably more demands on voters than the requirement that they only use weakly undominated strategies.

**Lemma 3.** Let  $K^\infty = \bigcup_{i \in N} B_i^\infty$ . Then  $|K^\infty| = 2$ .

*Proof.* Index the candidates such that  $j < l \iff x^j < x^l$ . Let  $\bar{k}^z = |\bigcup_{i \in N} B_i^z|$ . For all  $i \in N$ , either  $b_1$  or  $b_{\bar{k}^z}$  is weakly dominated by single peakedness. Eliminating weakly dominated actions,  $B^1$  is obtained. Let  $l^z = \arg \max_{j \in \bigcup_{i \in N} B_i^z} x^j$  and  $h^z = \arg \min_{j \in \bigcup_{i \in N} B_i^z} x^j$ . Let  $L^z = \{i \in N | l^z \in B_i^z\}$  and  $H^z = \{i \in N | h^z \in B_i^z\}$ . Then  $|L^1| + |H^1| = n$ , and by  $n$  odd, either  $|L^1| < (n+1)/2$  or  $|H^1| < (n+1)/2$ . W.l.o.g. assume the former. Then  $l^1 \notin B_i^2$ ,  $\forall i \in N$ , as candidate  $l^1$  cannot possibly win when over half the voters have eliminated the action of voting for him. When candidate  $l^1$  no longer has a chance to win, a vote for  $l^1$  is dominated under  $B^1$  (unless there are only two candidates remaining in  $B^1$ ). The above argument is then repeated, removing one candidate at a time from *all* the voters' action sets, until further elimination of dominated strategies is not possible. Then  $|B_i^\infty| = 1$ ,  $\forall i \in N$ , and  $|K^\infty| \leq 2$ . When only two candidates remain, casting a vote for one's favorite candidate cannot be dominated. Finally,  $|K^\infty| \neq 1$  because the candidates are voters as well. ■

<sup>23</sup>To reduce clutter I drop the dependence on ballot number and focus on a single voting game.

Lemma 3 states that the iterated elimination of dominated strategies reduces the set of actions to be consider for each voter to a singleton. Moreover, it is easy to see that each voter will cast their vote for one of two candidates; the Condorcet winner,  $k^*$ , or the candidate with the most similar platform on the left or the right (i.e.,  $k^* - 1$  or  $k^* + 1$ ). It goes without saying that  $k^*$  wins. Hence, under majority rule and single-peaked preferences, the iterated elimination of dominated strategies results in an unique equilibrium in which the Condorcet winner wins a majority of the vote on the first ballot.

The following proposition, which follows directly from Lemma 3, spells out the implications for equilibrium outcomes in the majority run-off game.

**Proposition 6.** Assume voters iteratively eliminate dominated strategies. If  $K^C = \{k^*\}$  then  $e_{k^*}^1 = 1$  and  $e_k^1 = 0, \forall k \in K \setminus k^*$ , is the unique equilibrium of the game.

The intuition behind the result is simple. First, the candidates' cost of running assures us that the game will not go on indefinitely. Backward induction logic can then be used to drive the result home. By Lemma 3, there are at most two candidates that receive votes at any terminal node and, furthermore, these candidates are the candidates whose platforms are closest to the median voter's ideal policy. Note that this rules out any multi-candidate equilibria as they can only be sustained by voter strategies that punish deviating candidates. The iterated elimination of dominated strategies rules this possibility out and, consequently, all the candidates, save the winner, will choose to exit as soon as possible. Finally, much like under sincere voting, a candidate that is not a Condorcet winner cannot win an outright victory.

To sum up, multiple non-centrist multi-candidate equilibria exist under strategic voting when voters use weakly undominated strategies. An unattractive quality of many of these equilibria is that they require the voters to solve a complex coordination problem (see the example above). Hence, in some sense a very high degree of rationality (and presumably effort) is required of the voters to sustain these equilibria – coupled with an absence of concern on part of the voters about how it may affect them. By refining our assumption about the voters' rationality, assuming that they iteratively eliminate dominated strategies, the possible equilibria are shown to be a subset of the possible equilibria under sincere voting.

## 5. CONCLUSIONS

The study of electoral coordination has not received due attention although it implicitly provides the theoretical backbone of the vast literature on electoral systems and the number of parties. As a result, the insights of the literature on political competition have been limited in some ways. On one

hand, the literature has sought to explain the relationship between electoral systems and the number of parties. On the other hand, it has considered how electoral competition influences the platforms adopted by political parties. The more realistic, but less frequently taken, approach is to address both questions simultaneously. To tackle both questions more efficiently a richer theory of electoral coordination is called for.

The model of electoral coordination presented here departs from the literature's conventional approach in a couple of important ways. First, the model is dynamic and electoral coordination is thus not assumed to occur instantaneously. Second, the candidates are allowed to make decisions whether to stay in the race or to exit. Hence, the political elite, as well as the voters, can engage in electoral coordination. These changes allow me to extend some of the best-known results of the literature on electoral competition and coordination, as well as to provide some important insights into the dynamics of electoral coordination.

More generally, this article suggests a new approach to modeling electoral competition by casting the process of electoral coordination as a war of attrition. The benefit of this approach is that it is dynamic and it calls greater attention to the pre-election stage in which much of the coordination takes place. It also shifts the focus away from the role of the voters in achieving electoral coordination to the candidates – although strategic voting was considered here as well. This is desirable because the candidates' stakes in the election can safely be assumed to be higher than the voters' stakes. Furthermore, Bulow and Klemperer's (1999) analysis of a war of attrition with multiple firms and multiple prizes suggests a connection between the war of attrition and electoral coordination. They find that the number of firms that does not exit immediately equals the number of prizes plus one. In the context of elections in single-member districts this result is equivalent to Duverger's Law. Moreover, it also corresponds with Cox's (1997)  $M + 1$  rule where  $M$  denotes the number of seats available – or the number of prizes in Bulow and Klemperer's (1999) terminology.<sup>24</sup>

I show the existence of pure strategy equilibria analogous to Duverger's Law and the median voter theorem in a dynamic model of candidate exit. The results thus simultaneously predict the number of candidates, or parties, and their ideological orientation. Other equilibria may exist but they depend on knife-edge conditions, which are unlikely to obtain in large voter populations. The model also gives rise to mixed strategy equilibria where coordination failure occurs and the outcome is inefficient in the sense that a Condorcet winner is not elected and a delay in selecting a winner may occur. It is thus not necessary to resort to modeling uncertainty to explain

<sup>24</sup>Naturally, Bulow and Klemperer's (1999) results are silent on the issue of candidate positioning, i.e., asymmetries among the candidates, because of their focus on competition between firms. The question is, however, an interesting topic for future research.

coordination failure.<sup>25</sup> The likelihood of coordination failure, and each candidate's exit, is shown to depend on the cost of running that the candidate faces.

The model allows a number of extensions for future research. Uncertainty could be incorporated into the model. Candidates might be uncertain about the voters' preferences, their competitors' costs of running, or their policy platforms. Voters might similarly face uncertainty about the candidates' policy platforms. As it turns out, uncertainty about the candidates' cost of running does not alter the results substantively. The other types of uncertainty appear more likely to generate interesting insights.

Considering different electoral institutions is also of interest. Primary elections and sequential primary elections, such as the U.S. presidential primaries, would be particularly interesting applications. The model could also be modified to study majority run-off systems, a class of electoral systems that has received limited attention despite its popularity.<sup>26</sup> One of the interesting issues here concerns the effects of the first ballot electoral threshold on the number of parties and minority representation, which has received considerable attention in the study of primary elections in the southern states of the U.S. (e.g., Canon, 1999).

The influence of the candidates' costs on their strategies raises some interesting questions regarding effects of campaign finances on political competition. More specifically, the extent of public financing of electoral campaigns may influence the possibility of multi-candidate equilibria. The relationship is, however, more intricate than one might expect. Although public funding reduces the candidates' opportunity cost it does not translate directly into a higher probability of coordination failure as each candidate's strategy is also affected by the other candidates' costs. Figures 2 and 3 show that if Candidate 3 faces high costs, Candidate 1 and 2's costs must also be high. If, on the other hand, campaigns are publicly financed, Candidate 1 and 2's costs of running will be low – requiring Candidate 3's cost to be even lower if a three-candidate equilibrium is to exist. If the candidates' ability to raise contribution depends on their electoral success, i.e., which in effect lowers their costs of running, then smaller candidates and parties are likely to be further disadvantaged.

In sum, it has been shown that some of the most basic, and the best known, results in the literature on electoral competition do not depend on strategic voting but can be obtained by modeling electoral contests as a war of attrition between candidates. That is, the model accounts for some well-known empirical regularities without making unreasonably restrictive

<sup>25</sup>Uncertainty about electoral strength undoubtedly plays a role in some cases but often coordination failures occur when the actors have good information about their electoral viability.

<sup>26</sup>Majority run-off elections are most frequently associated with presidential elections. They are, however, used for the election of numerous other offices, including legislative elections. Over 29 countries use the majority run-off for the election of its legislators (Birch, 2003).

assumptions about the actors' rationality. Naturally many questions about electoral coordination remain but, at the very least, the results here suggest that modeling electoral competition as a war of attrition is a promising way to proceed.

#### ACKNOWLEDGMENTS

I am grateful to Randall Calvert, John Duggan, Mark Fey, Christian Grose, Thomas Gschwend, Christopher Kam, and Alan Wiseman for their helpful comments. I thank the Political Institutions and Public Choice Program at Michigan State University for its support. I also acknowledge financial support from the Fulbright Foundation. All remaining errors are mine.

INDRIDI H. INDRIDASON

*University of Oxford and  
University of Iceland*

#### REFERENCES

- Alvarez, R. M. and J. Nagler, 2000, A new approach for modelling strategic voting in multiparty elections. *British Journal of Political Science* 30, 57–75.
- Austen-Smith, D., 1984, Two-party competition with many constituencies. *Mathematical Social Sciences* 7, 177–198.
- and J. S. Banks, 1988, Elections, coalitions, and legislative outcomes. *American Political Science Review* 82, 405–422.
- Besley, T. and S. Coate, 1997, An economic model of representative democracy. *Quarterly Journal of Economics* 112, 85–114.
- Birch, S., 2003, Two-round electoral systems and democracy. *Comparative Political Studies* 36, 319–344.
- Blais, A., 2002, Why is there so little strategic voting in Canadian plurality rule elections? *Political Studies* 50, 445–454.
- and I. H. Indridason, 2007, Making candidates count: the logic of electoral alliances in two round legislative elections. *Journal of Politics* 69, 193–205.
- Bulow, J. and P. Klemperer, 1999, The generalized war of attrition. *American Economic Review* 89, 175–189.
- Callander, S. and S. Wilke, 2002, Candidate motivation and electoral competition. Mimeo.
- Calvert, R. L., 1985, Robustness of the multidimensional voting model: candidate motivations, uncertainty, and convergence. *American Journal of Political Science* 29, 69–95.
- Canon, D. T., 1999, *Race, Redistricting, and Representation: The Unintended Consequences of Black Majority Districts* (University of Chicago Press, Chicago).
- Cox, G. W., 1997, *Making Votes Count: Strategic Coordination in the World's Electoral Systems* (Cambridge University Press, Cambridge, UK).
- De Sinopoli, F. and A. Turrini, 1999, A remark on voters' rationality in Besley and Coate model of representative democracy. CORE Discussion Paper No. 9927.
- Dhillon, A. and B. Lockwood, 2001, Multiple equilibria in the citizen candidate model of representative democracy. Mimeo.
- Downs, A., 1967, *Inside Bureaucracy* (Little, Brown, Boston, MA).

- Dutta, B., M. O. Jackson, and M. Le Breton, 2001, Strategic candidacy and voting procedures. *Econometrica* 69, 1013–1037.
- Duverger, M., 1954, *Political Parties: Their Organization and Activity in the Modern State* (Methuen, London).
- Farrell, J., 1987, Cheap talk, coordination, and entry. *RAND Journal of Economics* 18, 34–39.
- Feddersen, T. J., 1992, A voting model implying Duverger's Law and positive turnout. *American Journal of Political Science* 36, 938–962.
- Fey, M., 1997, Stability and coordination in Duverger's Law: a formal model of preelection polls and strategic voting. *American Political Science Review* 91, 135–147.
- Fudenberg, D. and J. Tirole, 1986, A theory of exit in duopoly. *Econometrica* 54, 943–960.
- Golder, S. N., 2000, A new theory of the dynamics of mainstream right parties in France. Presented at the annual meeting of the American Political Science Association.
- Groseclose, T., 2001, A model of candidate location when one candidate has a valence advantage. *American Journal of Political Science* 45, 862–886.
- Hendricks, K. and C. Wilson, 1985, The war of attrition in discrete time. C. V. Starr Working Paper R.R. No. 85-41.
- Hottelling, H., 1929, Stability in competition. *Economic Journal* 39, 41–57.
- Jenkins, J. A. and T. P. Nokken, 2000, The institutional origins of the Republican Party: spatial voting and the House Speakership Election of 1855–56. *Legislative Studies Quarterly* 25, 101–130.
- Kaminski, M., 2001, Coalition stability of multi-party systems: evidence from Poland. *American Journal of Political Science* 45, 294–312.
- Kapur, S., 1995, Markov perfect equilibria in an N-player war of attrition. *Economics Letters* 47, 149–154.
- Ledyard, J. O., 1984, The pure theory of large two-candidate elections. *Public Choice* 44, 7–41.
- Maynard Smith, J., 1974, The theory of games and the evolution of animal conflicts. *Journal of Theoretical Biology* 47, 209–221.
- Mitchell, D. W., 1987, Candidate behavior under mixed motives. *Social Choice and Welfare* 4, 153–160.
- Montgomery, K. A., 1999, Electoral effects on party behavior and development: evidence from the Hungarian National Assembly. *Party Politics* 5, 507–523.
- Moser, R. G., 2001, *Unexpected Outcomes: Electoral Systems, Political Parties, and Representation in Russia* (University of Pittsburgh Press, Pittsburgh, PA).
- Myatt, D. P., 2005, Instant exit from the asymmetric war of attrition. Discussion Paper No. 160, Department of Economics, University of Oxford.
- Myerson, R. B. and R. J. Weber, 1993, A theory of voting equilibria. *American Political Science Review* 87, 102–114.
- Osborne, M., 1995, Spatial models of political competition under plurality rule: a survey of some explanations of the number of candidates and the positions they take. *Canadian Journal of Economics* 27, 261–301.
- and A. Slivinski, 1996, A model of political competition with citizen-candidates. *Quarterly Journal of Economics* 111, 65–96.
- Osborne, M. J., 2000, Entry-detering policy differentiation by electoral candidates. *Mathematical Social Sciences* 40, 41–62.
- Palfrey, T. R., 1984, Spatial equilibrium with entry. *Review of Economic Studies* 51, 139–156.

- , 1989, A mathematical proof of Duverger's Law, in: P. C. Ordeshook, ed., *Models of Strategic Choice in Politics* (University of Michigan Press, Ann Arbor, MI) 69–92.
- Plott, C. R., 1967, A notion of equilibrium and its possibility under majority rule. *American Economic Review* 57, 787–806.
- Quinn, T., 2005, Leasehold or freehold? Leader-eviction rules in the British Conservative and Labour Parties. *Political Studies* 53, 793–815.
- Stewart, C. III., 1999, The inefficient secret: organizing for business in the U.S. House of Representatives, 1789–1861. Paper presented at the annual meeting of the American Political Science Association.
- Wittman, D., 1977, Candidates with policy preferences: a dynamic model. *Journal of Economic Theory* 59, 407–425.
- , 1983, Candidate motivation: a synthesis of alternative theories. *American Political Science Review* 77, 142–157.